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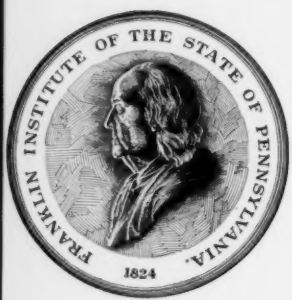

1956

Automatic Control Unit for Model Railroads

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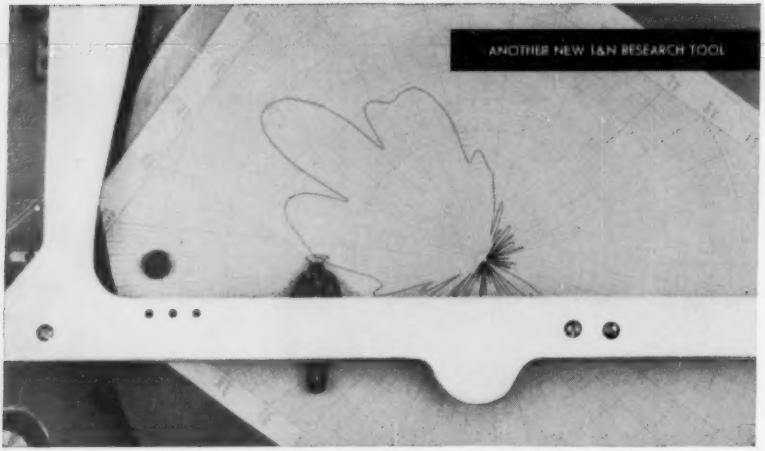

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INDEX TO ADVERTISERS

	PAGE
Ajax Electric Furnace Corp.	ii
Almo Radio Co.	vii
Atlas Photo Engraving Co.	xii
Bell Telephone Laboratories	viii
Bolger-Parker Co.	xii
Buten, M. & Sons	vii
Coleman, W. B., & Co.	xi
Damon & Foster	xi
Davidson, P. L.	xi
Eggly-Furlow Engineers	xi
Fehr & Johnson, Inc.	xii
Fidelity-Philadelphia Trust Co.	xiv
Franklin Institute, Awards	xiii
Franklin Institute, Laboratories	Fourth cover page
Franklin Institute, Membership	Third cover page
Harris-Dechant Associates	xi
Herbach & Rademan	xii
Hess & Barker	x
Kearney Lumber Co.	xii
Lancaster Press, Inc.	vi
Leeds & Northrup Co.	Second cover page
Leopold, Charles S.	xi
Levy & Co., Max	x
Shanahan & Co.	xii
Steinberg and Company, Albert	vii
Thomas Co., Arthur H.	x
Yarnall-Waring Co.	vi

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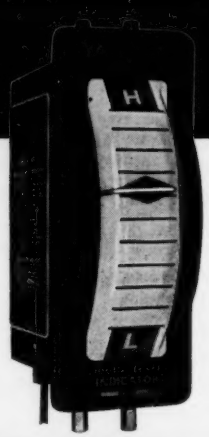

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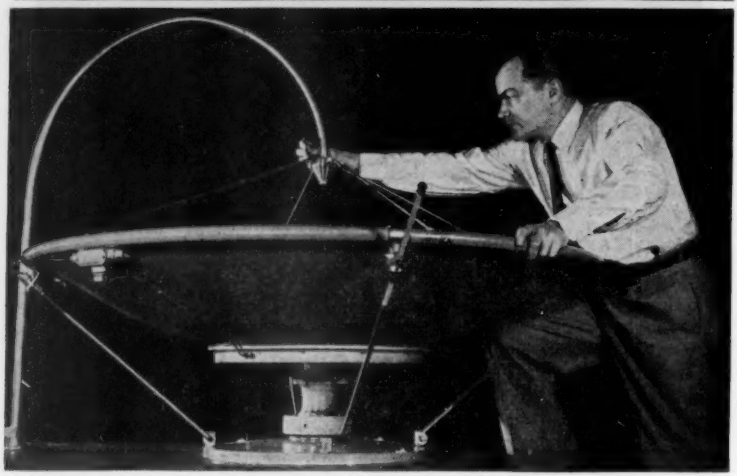

Bell Laboratories' Dr. J. W. Fitzwilliam adjusts a waveguide feed to a parabolic dish reflector. Dr. Fitzwilliam, who has a Ph.D. in physics from Massachusetts Institute of Technology, leads the practical development of Bell's new 11,000-mc. system. Components had to be specially developed to operate in a frequency band not previously utilized.

The giant microwave highway that carries your TV programs along with telephone conversations from coast to coast has a versatile new partner—an entirely new microwave system which was created, and is now being developed, at Bell Laboratories. The new system operates at 11,000 megacycles—a much higher frequency than ever before used in telephone service.

Bell's present microwave systems—operating at 4000 megacycles—were designed for heavy traffic and long distances. The new system is designed especially for lighter traffic and shorter distances—up to 200 miles. Its traffic capacity is extremely flexible. Depending on traffic needs, the system can provide only one one-way or as many as three two-way broadband channels. Each two-way channel can carry 200 telephone conversations simultaneously or one television program in color or black and white in each direction along a route. The new system, which is now being operated experimentally, will be valuable in providing additional telephone service and TV programs for cities in remote areas.

This is another example of how research and development work at Bell Telephone Laboratories help the Bell Telephone System to serve you better.

Mr. L. C. Tillotson, who originated the new system, adjusts the klystron-isolator combination which made the system feasible. Mr. Tillotson, an M.S. from the University of Missouri, heads research in microwave applications.

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Vol. 262 **OCTOBER, 1956** **No. 4**

CONTENTS

Automatic Control Unit for Model Railroads ..... JOHN STONE 247

Pressure Transmission in Liquid Films ..... EDWARD DAWSON 255

A Pursuit Problem ..... R. M. ROSENBERG 265

Reliable Circuits Using Less Reliable Relays, Part II ..... E. F. MOORE AND C. E. SHANNON 281

Notes from the National Bureau of Standards 299

The Franklin Institute
- Library 304
- Membership 307
- Journal of The Franklin Institute 308
- Notes from The Franklin Institute Museum 309
- Notes from The Franklin Institute Laboratories for Research and Development 311
- Notes from the Biochemical Research Foundation 313

Publications Received 316

Book Reviews 317

Book Notes 322

Current Topics 264, 280, 298, 323

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The Louis E. Levy Medal (1923—Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

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The Walton Clark Medal (1926—Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

The Frank P. Brown Medal (1938—Silver Medal).—This medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

The Newcomen Medal (1943—Gold Medal).—This medal is awarded, not oftener than once in three years, for achievement in the field of Steam.

The Francis J. Clamer Medal (1943—Silver Medal).—This medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

The Stuart Ballantine Medal (1946—Gold Medal).—This medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

The Boyden Premium (1859).—This premium is awarded not oftener than once in five years to any resident of North America who has recently made a notable experimental determination of the speed, in free space, of radiation in any region of the entire spectrum.

The William M. Vermilye Medal (1937—Bronze Medal).—This medal is awarded not oftener than biennially in recognition of outstanding contribution in the field of Industrial Management.

The Certificate of Merit (1882).—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

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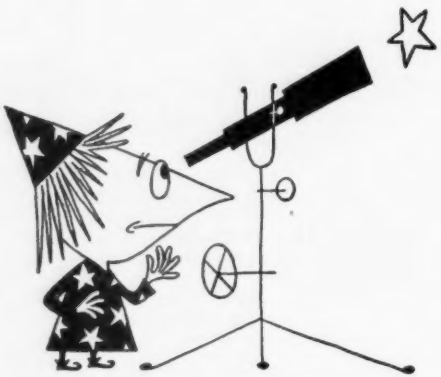

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Journal
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Vol. 262 OCTOBER, 1956 No. 4

AUTOMATIC CONTROL UNIT FOR MODEL RAILROADS

BY

JOHN STONE[1]

The Franklin Institute is building, in its Locomotive Room, a model railroad layout that will be as realistic in scenic beauty and performance as one could ever desire. As a locomotive winds its way about the several loops, it maintains a constant speed regardless of the number of cars it pulls or the steepness of the grades it climbs and descends. The unit that automatically controls the speed of the locomotive is a unique feature of this "dream layout."

The problem of keeping a slow freight moving on an upgrade without stalling presents a nice challenge to anyone who desires a completely automatic layout. Mr. Lynn H. Wescott, Editorial Director of *The Model Railroader Magazine*, submitted a theoretic electronic circuit which, after further development, solved this problem very nicely and easily. With this control, the operator sets a throttle, throws a switch to start the locomotive, and then just sits back and watches it accelerate realistically to the pre-selected speed. As a grade is approached, the voltage supplied to the track is automatically adjusted to compensate for the change in load so that the speed remains constant. Likewise, when the stop switch is thrown, the locomotive slows down to a full stop and power is automatically cut off from the track. An overload-detecting circuit limits the power-pack output to a safe value and, when the trouble is cleared, the locomotive speeds up to the pre-selected speed at which it was running before the overload occurred. Trackside block signals can be connected into the electronic unit to start, slow or stop a locomotive automatically and thus make possible a completely programmed operation.

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Automatic control is accomplished by a motorized, variable-voltage transformer whose output directly controls the voltage supplied to the track. A transformer (the type used in the output stage of an audio amplifier to drive a speaker) is connected in series with the power-pack and variable transformer output so that it has a high step-up ratio; that is, the "speaker" terminals are connected in series with the power-pack and variable transformer output, and the leads that are normally connected to the "plate" terminals are used to translate the current change in the locomotive motor to a voltage. This voltage and that of the variable transformer output are both rectified, and the two rectified voltages are connected in opposition to each other. The throttle voltage is then applied so that it adds to the rectified voltage from the current-sensing transformer. The purposes of these three voltages are:

throttle voltage determines the speed of the locomotive;
current-sensing voltage adds or subtracts from the throttle voltage to correct for change of loading on the locomotive motor; and
variable transformer voltage subtracts from the current-sensing voltage so that the voltage created by the load current under normal conditions can be cancelled out, leaving a zero signal at the grid of the electronic tube when the train is travelling at the desired speed on a level track.

A twin triode (6SL7) amplifies any difference between these voltages and controls a relay connected in the plate circuit of each triode. Either relay will cause the motorized variable transformer to drive until the difference in voltage at the grid of the tube is zero. Each relay is of the telephone type with 10,000-ohm coils. The speed of the motor used to drive the variable transformer must, as recommended by Mr. Wescott, be such that the locomotive will go from a full stop to maximum speed in 6 to 12 seconds. The motor speed that best meets this requirement is one of 2.5 to 3 rpm., and the rating is 1.5/1000 hp. The overload protective device is a relay connected in series with the output of the power-pack so that when the output current reaches a pre-selected maximum value, it will bias the twin triode. This drives the variable transformer to a lower voltage, and thus decreases the output current. The overload relay then regulates the output current to the value selected by adjustment of a shunt resistor on the relay coil. When the overload on the circuit is reduced, the bias is removed from the triode and the circuit returns to normal.

Control of this system can be made either semi-automatic or completely automatic. The semi-automatic system uses a toggle switch for the stop-go selection and another toggle switch for the reversing selection. The stop-go toggle switch disconnects the throttle voltage and removes the ground connection from the connector relay. The connector relay remains energized until the brake relay drops out just

as the locomotive stops. The connector relay then drops out and disconnects the power from the track. The reversing toggle switch is used to hold the reversing relay in after the connector relay has dropped out. The locomotive can therefore be stopped and started in the same direction as many times as desired without the reversing relay changing its setting. However, before the train can reverse, it must slow down to a complete stop and the stop-go switch must be turned to "go" again before the locomotive will proceed in the reverse direction.

TABLE I.

Relay Specifications

Coil Voltage	Coil Resistance	No. of Poles	Name	Time Delay
—	11,300 ohms	3 PDT	Brake	—
—	11,300 ohms	2 PDT	Accelerate	—
24 d.c.	1,000 ohms 100 μF shunt	6 PDT	Conr.	0.1 sec.
24 d.c.	1,000 ohms (copper slug) slow release	4 PDT	Revr.	0.1 sec.
1.5–9v. d.c.	10 ohms	1 PDT	OVRR	—
24 d.c.	1,000 ohms	1 PDT	SLOR	—
24 d.c.	1,000 ohms	2 PDT	SGR	—
24 d.c.	1,000 ohms 150 μF shunt	3 PDT	BAKR	0.15 sec.
24 d.c.	1,000 ohms 150 μF shunt	3 PDT	FWDR	0.15 sec.

Motor Specifications

Barber-Coleman Co. No. PYAZ-928-B-3, 115 v., 60 cps., 35 watts, 3 rpm.

Power Pack

Scintilla No. AT-746
Input 120 v.
Output 18 v., 6 A

High Voltage Unit
250 v. d.c. 50 ma
6.3 v. a.c. 2.5 A

Scintilla Rectifier No. AR-106
Bridge Type
Input 18 v.
Output 12 v., 6 A

Variable Transformer

Superior Electric Co. Type 10, 0.165 KVA
Line Voltage 120, 60 cps.

Current Sensing Transformer

Thordarson T22S78

A toggle switch is also provided to reduce the speed of the locomotive without changing the throttle setting. This is done by replacing the throttle voltage with another voltage of the proper value to bring about a reduced speed.

A completely automatic system can be attained by replacing these switches with relays activated by programming or track detection (signal) circuits. These relays are connected in such a way that the sequence of operations can be accomplished either by an impulse or an extended-period type of signal.

If desired, a meter can be connected to the output terminals of the power-pack to indicate either track voltage or current. A 2-ma. meter

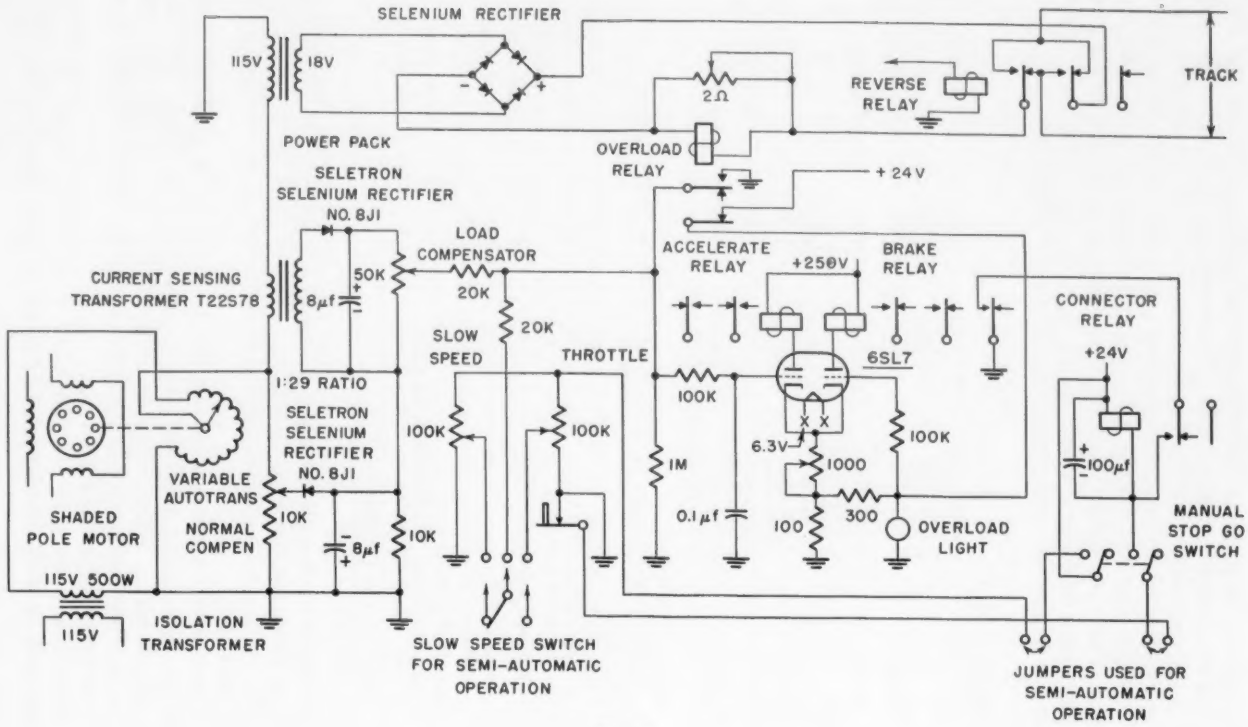
SELENIUM RECTIFIER
115V
18V
POWER PACK
2Ω
REVERSE RELAY
TRACK
OVERLOAD RELAY
+ 24V
SELETRON SELENIUM RECTIFIER NO. 8J1
LOAD COMPENSATOR
ACCELERATE RELAY
+250V
BRAKE RELAY
CONNECTOR RELAY
CURRENT SENSING TRANSFORMER T22S78
50K
8μf
20K
SLOW SPEED
20K
THROTTLE
6SL7
+24V
1:29 RATIO
100K
100K
100K
100K
1M
6.3V
1000
100μf
SELETRON SELENIUM RECTIFIER NO. 8J1
VARIABLE AUTOTRANS
NORMAL COMPEN
10K
8μf
10K
0.1μf
100
300
OVERLOAD LIGHT
SHADED POLE MOTOR
MANUAL STOP GO SWITCH
115V 500W
115V
ISOLATION TRANSFORMER
SLOW SPEED SWITCH FOR SEMI-AUTOMATIC OPERATION
JUMPERS USED FOR SEMI-AUTOMATIC OPERATION

FIG. 1.

is well suited for the application, since the current range is in the order of 1 ampere and the voltage range in the order of 10 volts.

The power supplies for the control consist of a 115-volt isolation transformer, a 24-volt d-c. power-pack and a 250-volt d-c. power-pack. Table I lists the specifications for these power supplies and all of the relays required.

Figure 1 shows the complete signal generating and sensing circuit

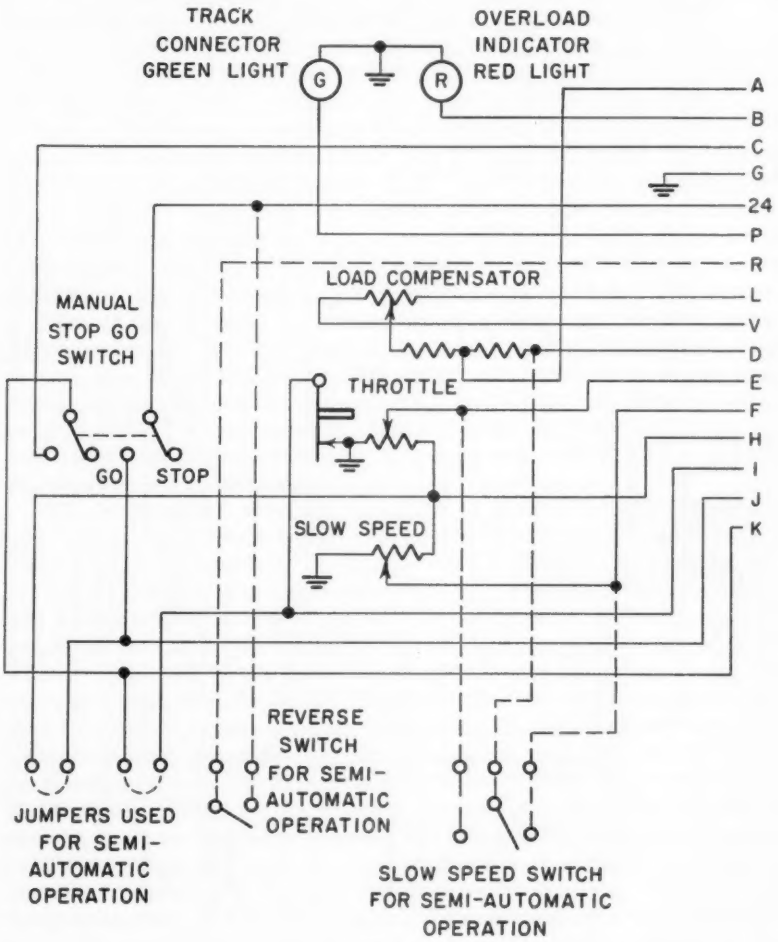

FIG. 2.

NOTE: Dotted lines are used when semi-automatic operation is desired. Leads *D, E, F, H, I, J, K* are not used for semi-automatic operation. All solid lines are used for completely automatic operation.

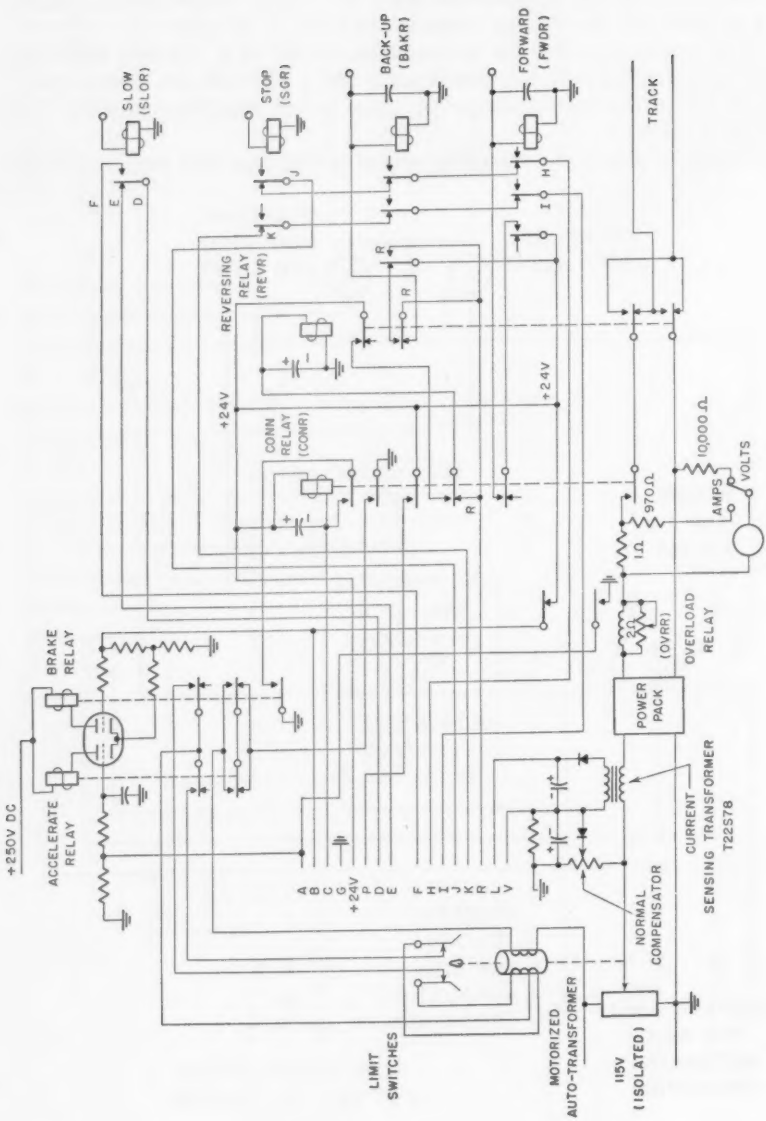
+250V DC
ACCELERATE RELAY
BRAKE RELAY
SLOW (SLOR)
STOP (SGR)
BACK-UP (BAKR)
FORWARD (FWDR)
REVERSING RELAY (REVR)
CONN RELAY (CONR)
+24V
TRACK
10,000Ω
AMPS
VOLTS
METER
970Ω
1Ω
OVERLOAD RELAY
(OVRR)
POWER PACK
CURRENT SENSING TRANSFORMER T22S78
NORMAL COMPENSATOR
MOTORIZED AUTO-TRANSFORMER
115V (ISOLATED)
LIMIT SWITCHES
A B C G +24V P D E F H I J K R L V

FIG. 3.

and its circuit values. Figure 2 is a diagram of the control panel and indicates the connections required for both semi-automatic and completely automatic operation. Figure 3 shows the complete motorized throttle with connections of all the relays required for completely automatic operation. The SLOR, SGR, BAKR and FWDR relays are omitted for semi-automatic operation. The resistance components in the meter (2 ma. full scale) circuit are 1 ohm in series with the line and 970 ohms in series with the selector switch. A resistance of 10,000 ohms is in series with the voltage terminal on the selector switch.

The circuit, when connected for semi-automatic operation, will follow a definite sequence of operation when the slow, start or stop switches are operated. When the manual "stop-go" switch is moved from "go" to "stop," the +24 v. is removed from the throttle potentiometer and a ground connection is removed from the "connector" relay. A braking signal is now received at the twin triode and the braking relay will pull in, driving the motorized variable transformer toward zero voltage. When the braking relay pulls in, it also makes a ground connection to the connector relay and holds it in thus allowing the train to slow down gradually. As the locomotive comes to a stalling point, the current increase in the sensing transformer gives a signal to the twin triode which causes the "brake" relay to drop out and the "accelerate" relay to pull in. When the brake relay drops out, it disconnects the only remaining ground on the connector relay and allows the connector relay to drop out. This disconnects all power from the track and stops the motorized variable transformer. The same sequence is followed when the throttle is turned to its complete "off" position.

The completely automatic operation is similar but the slow-speed and reverse manual switches are replaced by relays, with a stop, back-up and forward relay added. The slow relay must remain energized for the duration of the slow speed operation. De-energized, the locomotive will be controlled by the throttle. The stop relay also must remain energized for the period during which the locomotive is to be stopped. The back-up relay locks itself in and disconnects the ground from the connector relay and the +24 v. from the throttle. The train then will slow to a full stop; the connector relay will drop out; the reversing relay will pull in; and, when the back-up signal is removed, the train will start up slowly in the reverse direction until it reaches the speed selected by the throttle. When a stop signal is received and the train is going in reverse, the train will slow to a stop and the reversing relay will remain in the reversing position. A "go" signal will then start the train in reverse, accelerating to the speed selected by the throttle. The forward relay locks itself in and disconnects (1) the ground from the connector relay, (2) the 24 v. from the throttle, and (3) the holding circuit on the reversing relay. The train will slow to a full stop, the connector relay will drop out, the reversing relay will drop out and, when the forward

signal is removed, the train will start up slowly in the forward direction accelerating until it reaches the speed selected by the throttle. This sequence just described shows that before the train can change direction, it must first slow to a complete stop, and then accelerate slowly in whichever direction is selected.

The motorized throttle circuit can be adjusted by shorting the 0.1 microfarad capacitor at the grid of the twin triode and then adjusting the 1000-ohm rheostat in the cathode circuit until both relays in the plate circuits have just dropped out. Remove the short across the capacitor and, with a locomotive on the track, advance the throttle to full open, the "Load Compensator" to zero and the "Normal Compensator" to maximum. When the locomotive has reached full speed, turn the throttle quickly to the off position making sure that the switch on the throttle potentiometer has opened. Just as the locomotive is coming to a stop, advance the load compensator potentiometer to the point where the connector relay drops out. The "Overload" relay is best adjusted by having an ammeter in series with the power-pack output. Adjust the shunting rheostat to limit the current to a value that will not damage the power-pack when a short-circuit appears at the track.

With assembly and adjustment of the motorized throttle unit completed, the model railroader now has a power system that will provide truly realistic operation.

PRESSURE TRANSMISSION IN LIQUID FILMS

BY

EDWARD DAWSON [1]

SUMMARY

When a liquid is subjected to variable pressure small amounts of flow occur because of its compressibility, and in thin clearance spaces between solid boundaries the transmission of a change in pressure may be appreciably delayed by viscous resistance to this flow. The magnitude of this delay is calculated for assumed theoretical conditions of smooth, parallel, and rigid boundary surfaces, and clean liquid having constant viscosity and bulk modulus. Experiments are described in which the sliding of steel blocks in an oil-filled chamber was temporarily stopped by conditions of hydraulic unbalance and friction resulting from delay in the transmission of pressure increments.

I. MATHEMATICAL THEORY OF PRESSURE TRANSMISSION IN A VISCOUS AND COMPRESSIBLE LIQUID BETWEEN PARALLEL PLANES

The solid boundaries, Fig. 1, are located in the planes $y = 0$ and $y = h$, and it is assumed that they are of infinite width perpendicular

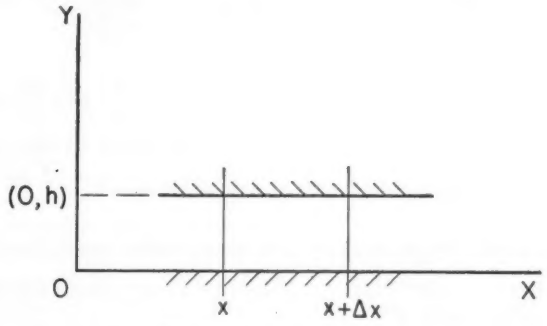

FIG. 1. Boundaries of film.

to the XY-plane. The volume of a rectangular element between sections at x and $x + \Delta x$, and of width b perpendicular to the XY-plane, is

$$v_1 = b\,h\,\Delta x. \tag{1}$$

The rate of change of this volume is

$$\frac{dv_1}{dt} = b\,\frac{dh}{dt}\,\Delta x. \tag{2}$$

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If ρ denotes the mass of a unit volume of the liquid at pressure p, the mass of liquid in a volume v at pressure p is

$$M = \rho v. \tag{3}$$

If ρ is a function of time, the volume occupied by a constant mass, M, of liquid changes at the rate

$$\frac{dv}{dt} = M\frac{d}{dt}\left(\frac{1}{\rho}\right) = -\frac{M}{\rho^2}\frac{d\rho}{dt}. \tag{4}$$

The bulk modulus of the liquid, E, is defined by

$$E = \rho\frac{dp}{d\rho}. \tag{5}$$

Hence

$$\frac{d\rho}{dt} = \frac{\rho}{E}\frac{dp}{dt}, \tag{6}$$

and, from Eqs. 3 and 4,

$$\frac{dv}{dt} = -\frac{M}{\rho^2}\frac{d\rho}{dt} = -\frac{M}{\rho E}\frac{dp}{dt} = -\frac{v}{E}\frac{dp}{dt}. \tag{7}$$

By Eq. 2 the rate of change of the space v_1 is $\frac{dv_1}{dt} = b\frac{dh}{dt}\Delta x$, and by Eq. 7 the rate of change, due to pressure variation, of the volume occupied by a mass of liquid of instantaneous volume v_1 is $\left(\frac{dv}{dt}\right)_{v=v_1} = -\frac{v_1}{E}\frac{dp}{dt}$. In general, these rates are not equal and a compensating difference exists between the volumes of liquid flowing across the sections at x and $x + \Delta x$ of amount

$$\frac{\partial Q}{\partial x}\Delta x = -\left(b\frac{dh}{dt}\Delta x + \frac{v_1}{E}\frac{\partial p}{\partial t}\right). \tag{8}$$

On inserting the value of v_1 from Eq. 1 and dividing both sides by Δx, Eq. 8 reduces to

$$\frac{\partial Q}{\partial x} = -\left(b\frac{dh}{dt} + \frac{bh}{E}\frac{\partial p}{\partial t}\right). \tag{9}$$

Michell (1)[2] shows that the volume rate of flow produced by a

[2] The boldface numbers in parentheses refer to the references appended to this paper.

pressure gradient $\frac{\partial p}{\partial x}$ in a viscous liquid between parallel planes is

$$Q = -\frac{bh^3}{12\,\mu}\frac{\partial p}{\partial x}, \tag{10}$$

where μ is the coefficient of viscosity. Hence

$$\frac{\partial Q}{\partial x} = -\frac{bh^3}{12\,\mu}\frac{\partial^2 p}{\partial x^2}, \tag{11}$$

and, from Eqs. 9 and 11,

$$\frac{\partial^2 p}{\partial x^2} = \frac{12\,\mu}{h^2}\left(\frac{1}{h}\frac{dh}{dt} + \frac{1}{E}\frac{\partial p}{\partial t}\right). \tag{12}$$

This is a general equation for the transmission of pressure through a viscous compressible liquid between parallel planes separated by a variable distance, h. It is applicable when h is small enough, in relation to the viscosity and compressibility of the liquid, for inertia effects to be neglected. When h is constant it reduces to

$$\frac{\partial p}{\partial t} = k\frac{\partial^2 p}{\partial x^2} \tag{13}$$

where

$$k = \frac{Eh^2}{12\,\mu}.$$

Equation 13 is analogous to the partial differential equation for the linear flow of heat in a solid bounded by two parallel planes, and many solutions, for particular end conditions, are given in works on the conduction of heat (2). The pressure, p, corresponds to temperature, and the coefficient, k, to the diffusivity of the material.

A condition of particular interest is that in which the region $-L < x < L$ is at zero pressure initially, and a constant pressure, P, is maintained at $x = \pm L$ when $t > 0$. A solution of Eq. 13 which satisfies these conditions is

$$p = P - \frac{4P}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}e^{-k(2n+1)^2\frac{\pi^2 t}{4L^2}}\cos\frac{(2n+1)\,\pi\,x}{2L}. \tag{14}$$

On introducing the dimensionless variables $T = \frac{kt}{L^2}$ and $X = \frac{x}{L}$, Eq. 14 reduces to

$$\frac{p}{P} = 1 - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}e^{-(2n+1)^2\frac{\pi^2 T}{4}}\cos\frac{(2n+1)\,\pi\,X}{2}. \tag{15}$$

By symmetry, Eq. 15 is also applicable to the region $0 < x < L$, with zero initial pressure, constant pressure P at $x = L$ for $t > 0$, and no flow across the section $x = 0$. This gives the rise of pressure in a rectangular slot of infinite width, depth L, and thickness h, on sudden application of a constant pressure P at its open end. Values of $\frac{p}{P}$ for various values of X and T may be obtained from graphs in (2). Graphs of $\frac{p}{P}$ as a function of T for various values of X, derived from this source, are given in Fig. 2.

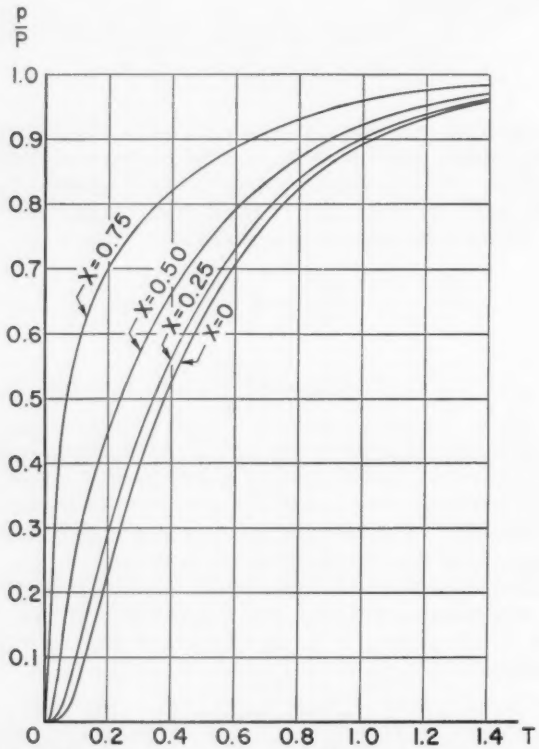

FIG. 2. Graphs of pressure variation in liquid between parallel planes.

The time required for the pressure between the boundary planes to increase from zero to any specified fraction of the suddenly applied external pressure P can be readily calculated from these results. Using, for example, the values of viscosity and bulk modulus in Table I for water and a light lubricating oil, the time in which the pressure at the

TABLE I.

Viscosity, μ $\frac{\text{dyne sec.}}{\text{cm}^2}$		Bulk Modulus, E $\frac{\text{dyne}}{\text{cm}^2}$	
Water	Oil	Water	Oil
0.01	0.25	$2.0(10)^{10}$	$2.0(10)^{10}$

root of an infinitely wide slot 3 centimeters deep would increase from zero to 0.5 P is given in Table II for various values of the slot thickness, h.

Solutions for conditions of sinusoidal variation of the external pressure P may also be obtained from analogous solutions for heat conduction (2). The pressure is then equal to the sum of a steady-state pulsation and a damped transient term.

The microscopic solids present in ordinary liquids would tend to retard flow through the film and further delay the transmission of pressure. Quantitative information on the number and size of these solids is seldom available, although various methods of filtration may be used to remove particles which exceed specified dimensions. Filters for lubricating oils are often provided to remove particles larger than 5 mi-

TABLE II.—*Time Required for Pressure at $x = 0$ to Increase from Zero to 0.5 P for $L = 3$ cm. and Various Values of h.*

h (centimeters)		0.0001	0.00005	0.00001	—	—	—
h (Angstroms)		10,000	5,000	1,000	200	37.7	25
t, sec.	oil	0.05	0.2	5.1	128	3600	8180
	water	0.002	0.008	0.2	5.1	144	327

crons (0.0005 cm.) in diameter, and filters of special design can remove particles down to 0.5 micron (5000 Å). Nevertheless, large numbers of much smaller solids, from 10 Å in diameter upward, are present in most liquids. These do not settle, but execute the Brownian movement due to momentary unbalance of molecular impacts against them. When flow takes place in a thin clearance space they may become wedged in the slender passages between the comparatively rough boundaries, with the obstructed larger particles stopping smaller ones in a cumulative manner. The transmission times given in Table II are lower limits that would occur with perfectly clean liquids and smooth boundary surfaces.

II. EXPERIMENTS ON SLIDING OF STEEL BLOCKS IN OIL

The theoretical results of Section I show that the time required for pressure changes to be transmitted through liquids in thin clearance spaces can be of appreciable magnitude. Consequently, an experimental investigation was made of the effect of delay in pressure trans-

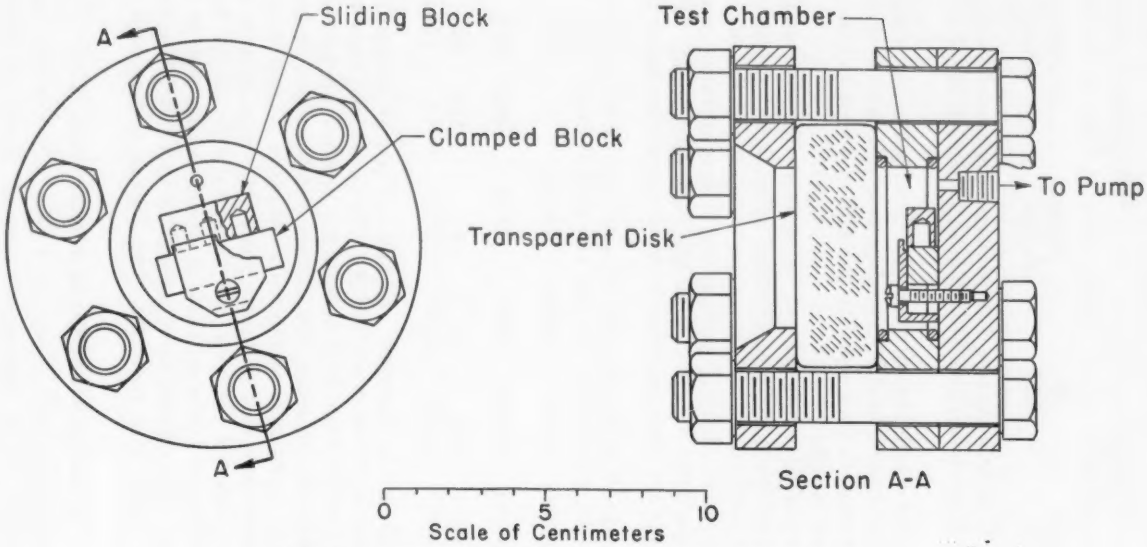

FIG. 3. Apparatus for experiments on sliding of steel blocks in oil.

mission on the resistance to sliding of metal blocks completely immersed in oil. The apparatus consisted of a cylindrical test chamber containing two rectangular blocks, the upper block being free to slide upon the lower one, which was clamped to a vertical wall of the test chamber, as shown in Fig. 3. The chamber was filled with mineral oil; and a hand pump, reservoir, and valves were provided so that the pressure of the oil in the chamber could be varied between atmospheric pressure and 200 kg. per sq. cm. The pump and the test chamber were connected by a length of flexible hydraulic hose.

In the first experiments, the blocks were made of cold-drawn steel 9.5 mm. by 12.7 mm. in cross section. One 9.5 mm.-wide face of each block was made flat and smooth by lapping until the blocks would "adhere" when wrung together, although the adherence was not as strong as it is with gauge blocks and the surface finish was not as smooth. The lower block, which was clamped to the test chamber with its flat surface upward, was 38 mm. long. Four sliding blocks were made, each of which was 28.6 mm. long. Cavities of different sizes were drilled in the flat face of each sliding block before it was lapped. The first block had three cavities, 4.8 mm. in diameter and 9.5 mm. deep, spaced 6.4 mm. center to center; the second had three cavities, 4.8 mm. in diameter and 1.7 mm. deep at the conical point made by the drill, spaced 9.5 mm. center to center; the third had 16 conical cavities, 2 mm. in diameter and 1 mm. deep, spaced 3.2 mm. center to center in two rows; and the fourth had a single conical cavity, 2 mm. in diameter and 1 mm. deep, at the center of its face.

In performing an experiment, one of the sliding blocks was placed upon the clamped block in the oil-filled test chamber, with the flat faces adjacent and with care being taken to see that the cavities in the sliding block were filled with oil. When the test chamber was closed, the upper block was made to slide along the lower block by slightly tilting the apparatus, and its motion was observed through a window of acrylic plastic material. The weight of the sliding block slowly forced oil from the space between the flat surfaces until solid support was established at a few high spots of the surfaces or through solid particles entrained in the oil. Under these conditions, the friction was partly solid and partly viscous, and its magnitude was such that sliding could be initiated and maintained by tilting the blocks to an angle of approximately 15 degrees from horizontal. When the hand pump was operated rapidly to increase the pressure in the test chamber during the sliding of the upper block, the sliding would stop suddenly and the upper block would stick to the lower one for a length of time which depended on arbitrary conditions of the test.

The sliding is stopped because of the time required for the pressure in the film between the blocks to increase to that of the surrounding liquid. During this delay the deficit of the oil pressure in the film ne-

cessitates an increase in the compressive stress at points of solid-to-solid contact. This increase in loading and stress at the solid junctions between the blocks is accompanied by an increase in friction, and this increased friction causes the blocks to stick. The effect is accentuated by the presence of localized cavities or indentations in the otherwise smooth surface of a block, since they increase the amount of oil that must flow through the film before pressure equalization is established. For example, when the first block with its three cavities of 4.8-mm. diameter and 9.5-mm. depth was used, a pressure increase of 18 kg. per sq. cm. would stop its sliding. When the pressure was rapidly increased to 100 kg. per sq. cm. and the apparatus was turned to place the plane of sliding in a vertical position, the block did not begin to slide until three minutes had elapsed. When the pressure was increased from 100 to 130 kg. per sq. cm., the downward motion was stopped but started again one minute later. Similar behavior was observed with the blocks having cavities of smaller volume, but the effect was less pronounced and the sliding could not be stopped at a vertical plane of sliding. However, at moderate angles of inclination, the sliding could be repeatedly stopped by rapidly increasing the test chamber pressure and started by reducing it, regardless of the volume of the cavities.

It is necessary to use filtered oil in these experiments, since comparatively large solid particles in the oil will hold the sliding surfaces far enough apart to make the delay in the equalization of pressure too brief to cause sticking. The adjacent surfaces of the blocks must also be smooth and accurate if the space between them is to be thin enough for sticking to occur. These experiments, consequently, provide an instance in which sticking is dependent upon conditions of sufficient accuracy of the sliding surfaces and cleanness of the lubricant.

To see whether sticking would occur with very smooth and flat surfaces in which no indentations had been drilled, further experiments were performed with two hardened-steel gauge blocks. The height of the sliding block was 10 mm., that of the clamped block was 16.5 mm., and both blocks were 35 mm. long and 9 mm. thick. The apparatus was tilted 16 degrees to start sliding of the upper block and when the pressure in the test chamber was rapidly increased to 170 kg. per sq. cm. the sliding stopped. The pressure was gradually reduced to 100 kg. per sq. cm. in an interval of 40 minutes but the block did not start to move. The pressure was then rapidly decreased to 70 kg. per sq. cm. and the upper block moved a distance of approximately 1 mm. and stopped. When the pressure was then reduced to atmospheric, the block slid to the end of its travel. In the next experiment the blocks were tilted 18.5 degrees and a rapid increase in pressure to 140 kg. per sq. cm. stopped the sliding, but it started again 12 seconds later. When the blocks were tilted 22 degrees the sliding was stopped for 7 seconds by rapidly increasing the pressure to 140 kg. per sq. cm.

These experiments show that transient conditions of hydraulic unbalance, with consequent increase in stress and friction at points of solid junction, can be caused by a delay in the transmission of pressure through a liquid in a thin clearance space. If the increase in pressure is slow, the effect may be expected to be negligible; on the other hand, more rapid pressure increases than those produced by a hand-operated pump in these experiments might be considerably more effective in causing unbalance and sticking.

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"Eye" for Detecting Gamma Rays.—A new "eye" in the form of a sodium iodide crystal in a watertight, windowless aluminum container is providing General Electric Company engineers at the Hanford atomic plant a better "look" into the dark depths of test wells located near atomic waste disposal grounds.

Suspended by cable and lowered into narrow test wells, the "eye" detects gamma rays present in the surrounding soil and transmits this information to a strip chart recorder located above ground.

Until the new deep well probe was introduced as a tool of Hanford's Radiological Sciences Department, an accurate accounting of what happened to such isotopes as radioactive cesium, ruthenium and antimony when dumped into the soil was extremely difficult.

Only when and if they reached the ground water level at 200 ft. or more where they could be tapped by water sampling could they be traced with ease.

Test wells drilled to depths of hundreds of feet are sampled at regular intervals at Hanford disposal ground to obtain data concerning the movement of ground wastes, dumped into the soil or held in underground tanks, depending on their concentrations of radioactive materials.

Long-lived isotopes, the kind which are too "hot" for too long to be deposited in the soil, are contained in sturdy tanks. Low-level wastes are permitted to be absorbed by the soil.

The "eye" fits in because it can detect gamma radiation at every point of its descent into the test well. What it "sees" and the depth at which it is "seen" is recorded on instruments above ground. The "eye" detects gamma activity and determines by energy spectrum analysis which isotopes are contributing to radiation. As a result, an adequate determination of the energies, quantities and soil effects of waste solutions is determined.

The unit was developed by members of the General Electric Radiological Sciences Department's instrument development unit at Hanford.

Hanford is operated by the General Electric Company for the Atomic Energy Commission.

New Life Saving Device.—A new life saving device that automatically inflates a pilot's "Mae West" life jacket upon contact with water has been perfected by North American Aviation, Incorporated. The automatic life jacket inflator compares in importance with such standard safety features as the automatic lap belt and automatic parachute release.

The search for an automatic inflator was triggered last year when North American Test Pilot George F. Smith became the first man in history to survive after bailing out of a plane at supersonic speed at low altitude. Smith landed unconscious in the Pacific Ocean off Laguna Beach, Calif., and would have drowned if men in a nearby boat hadn't pulled him from the water 50 seconds after he hit. His life jacket, which had to be manually operated, was not inflated. Had his Mae West been equipped with the new North American automatic inflator, he would have been assured of staying afloat, and his chances of survival would have been immeasurably increased had no one been nearby as he struck the water.

A PURSUIT PROBLEM *

BY

R. M. ROSENBERG [1]

INTRODUCTION

Wilder[2] has discussed the following problem: What will be the trajectories (in a horizontal plane) of a man walking with known constant speed such that he will continually keep a tree trunk between himself and an automobile which travels with known constant velocity along a straight road? The distance between tree trunk and road is known. In a paper comparing different doctrines of guiding missiles toward interception with a nonevading target, Magnus[3] has observed that Wilder's problem is equivalent to missile "pursuit curves with lead" in which the tangent of the missile trajectory converges to the virtual impact point of target and projectile under uniform linear velocity of both.

Ours is a generalization of Wilder's problem. We wish to find that trajectory which satisfies Wilder's problem (as described by Magnus) when the velocities of projectile and interceptor coincide and under the added conditions that (*a*) one of the position coordinates of the target (called henceforth "game") is unknown and (*b*) the vision of the interceptor (called henceforth "hunter") is severely restricted. The first of these is equivalent to regarding the distance between tree and road in Wilder's problem as unknown. To describe the second we state that the hunter can, at any instant, see only one point in space. This point lies in the intersection of the hunter's "line of sight," that is, the direction in which he looks—with a sphere about his position of radius equal to his "range of sight."

The condition to be described is that the hunter's range of sight depends linearly on that one of the hunter's position coordinates which is directed along the same axis as the unknown game coordinate. For instance, if it is the game's altitude which is unknown, then the hunter's range of sight is linearly proportional to the hunter's altitude. This second condition, not typical for hunters in general, is descriptive of a rather obvious method of operating airborne radars so as to eliminate "ground clutter." It consists in restricting the radar so as to accept only signals which are returned from objects nearer the radar by a predetermined constant than the ground.

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* This paper was originally written in 1951. Publication had to await security clearance.

[1] Professor of Applied Mechanics, University of Toledo, Toledo, Ohio.

[2] C. E. WILDER, *Am. Math. Monthly*, Vol. 38, pp. 17–25 (1931).

[3] W. MAGNUS, Translation Report No. F-TS-4026-RE, AMC, 1944.

It will be noticed that conditions (*a*) and (*b*) are both restrictive on the hunter. The first introduces an ambiguity into the position of the game, the second limits the hunter's ability to search for, or to see the game.

THE TWO-DIMENSIONAL PROBLEM

Consider the two-space shown in Fig. 1. The hunter is located at $H(y, z)$, the game at $G(\eta, \zeta_0)$ where ζ_0 is an unknown constant. We propose that, at any instant, the hunter can see only one point in space.

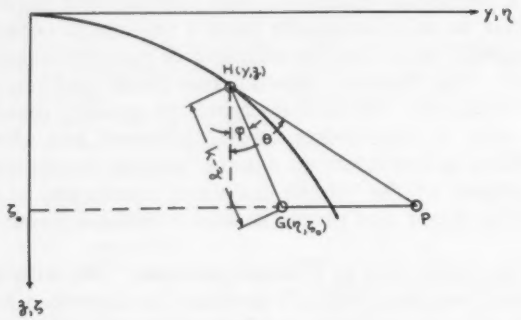

FIG. 1.

This point is the intersection of his line of vision with a circle about H of radius $k - z \geqslant 0$ where $k > 0$ is a constant. For $z > k$, the hunter becomes and remains blind. Let the curved line in Fig. 1 passing through $(0, 0)$ and $H(y, z)$ be the desired trajectory. Clearly, the hunter can see the game only if he looks in the direction of G, that is, when his line of sight makes the correct "sight angle" φ with the z-axis, and, if the distance between H and G is $k - z$.

It is one of the conditions of the problem that the game proceed along $\zeta = \zeta_0$ where ζ_0 is unknown (for the present we restrict the treatment to $|\zeta_0| < |k|$, with the constant velocity g and, that the hunter travel with the constant speed h. We introduce the condition $|h| > |g|$ because it insures that there always exists a point P on the line $\zeta = \zeta_0$ at which hunter and game can arrive simultaneously, provided the hunter leaves his curved trajectory at the instant when he sees the game and proceeds along the straight line connecting $H(y, z)$ with P. Later, we shall satisfy the condition that this straight line be tangent to the curved trajectory at (y, z).

The kinematical equations of motion of the hunter are

$$\left.\begin{aligned} \dot{y} &= h \sin\theta \\ \dot{z} &= h \cos\theta \end{aligned}\right\} \tag{1}$$

where θ is the angle between the tangent to the trajectory and the z-axis. We term it the "flight angle."

Inspection of Fig. 1 shows that the position of the game is

$$\left.\begin{aligned} \eta &= y + (k - z)\sin\varphi \\ \zeta &= \zeta_0 \end{aligned}\right\}. \tag{2}$$

Accordingly, we find the equations of motion of the game as

$$\left.\begin{aligned} \dot{\eta} &= g = \dot{y} + \frac{d}{dt}[(k - z)\sin\varphi] \\ \dot{\zeta} &= 0 \end{aligned}\right\}. \tag{3}$$

The conditions that hunter and game have a common future position at P, and that the line connecting $H(y, z)$ with P be tangent to the hunter's trajectory at (y, z) are summarized under the "pursuit principle," deducible from the two-dimensional velocity space, Fig. 2. We

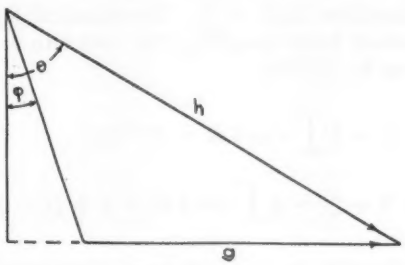

FIG. 2.

write it in the alternate forms

$$\left.\begin{aligned} \tan\varphi &= \tan\theta - c/\cos\theta \\ \sin\varphi &= (\sin\theta - c)/\sqrt{c^2 - 2c\sin\theta + 1} \end{aligned}\right\} \tag{4}$$

where $c = g/h < 1$.

The substitution of (1) and the second of (4) in the first of (3) gives the equation of the desired trajectory as

$$c = \sin\theta + \frac{d}{dt}\left[\left(\frac{k}{h} - \int_0^t \cos\theta\, dt\right)\frac{\sin\theta - c}{\sqrt{c^2 - 2c\sin\theta + 1}}\right]. \tag{5}$$

Let the initial instant be defined as $t = 0$ when $\eta = k$, $y = z = 0$ and assume that for $t < 0$ the hunter has travelled along the line $z = 0$; then, the initial conditions of the problem are $y(0) = z(0) = 0$, $\theta(0) = \pi/2$. It follows that the two-dimensional problem is now completely

formulated. The integral of (5) with the initial condition on θ yields $\theta(t)$ and, subsequently, the trajectory coordinates may be found from (1) by quadrature.

The unpalatable form of (5) is the direct result of a straightforward application of the visible relations governing the problem. Fortunately, it was observed that (5) becomes tractable through the transformations

$$\lambda = 1 - \frac{h}{k}\int_0^t \cos\theta\, dt$$

$$\mu = 1 + \frac{g}{k}t - \frac{h}{k}\int_0^t \sin\theta\, dt. \tag{6}$$

In the $\lambda\mu$-plane, (5) goes over into

$$\frac{d\lambda}{d\mu} = \sqrt{\left(\frac{\lambda}{\mu}\right)^2 - 1} \tag{7}$$

with the initial condition $\lambda(1) = 1$. The physical significance of the transformations which have simplified the problem in this spectacular manner is best seen by writing

$$k\lambda = k - h\int_0^t \cos\theta\, dt = k - z(t)$$

$$k\mu = k + gt - h\int_0^t \sin\theta\, dt = k + gt - y(t).$$

It is self-evident that $k\lambda$ is the distance between hunter and game (see Fig. 1). The quantity $k\mu$ is the y-component of that distance because, at $t = 0$, this distance was k; at any $t_i > 0$ the game has proceeded to $k + gt_i$ and the y-component of the hunter's position is $y(t_i)$. We note here one of the interesting features of this problem; it becomes elementary through the introduction of nonorthogonal coordinates. In fact, λ and μ are oblique coordinates whose degree of obliquity depends on time.

The integral of (7) is

$$\mu^2 = \left(\frac{\lambda}{\mu} + \sqrt{\left(\frac{\lambda}{\mu}\right)^2 - 1}\right) \exp\left\{-\frac{\sqrt{(\lambda/\mu)^2 - 1}}{\lambda/\mu - \sqrt{(\lambda/\mu)^2 - 1}}\right\} = f\left(\frac{\lambda}{\mu}\right) \tag{8}$$

and the $\lambda\mu$-correspondence is obtainable by simple computations. It is tabulated for certain values of λ/μ in $1 \leqslant \lambda/\mu \leqslant \infty$ in Table I. For

$\lambda/\mu < 1$, no real values of μ exist. The origin in the $\lambda\mu$-plane corresponds to that point at which $z = k$, that is, that point at which the hunter becomes blind. We shall call t_1 the time at which this occurs so that $z(t_1) = k$. Consequently, the relation (8) is applicable in $0 \leqslant t \leqslant t_1$ or in $0 \leqslant z \leqslant k$. It can easily be ascertained that the curve in the $\lambda\mu$-plane defined by (8) enters the origin tangent to the λ-axis, that is, $d\lambda/d\mu = \infty$ at the origin. But the transformations (6) imply $d\lambda/d\mu = \cos\theta/(\sin\theta - c)$; therefore, at t_1 we must have $\theta(t_1) = \sin^{-1} c$. Furthermore, the relation between θ and φ (see (4)) gives $\varphi(t_1) = 0$ which implies that $\eta(t_1) - y(t_1) = 0$ or, at t_1 the hunter and game are on one line parallel to the z-axis. For instance, if z denotes the vertical direction we conclude that, at $t = t_1$, the game is vertically below the hunter. For this particular situation, the straight line with inclination

TABLE I.

λ	μ
1.00000	1.00000
0.95938	0.87216
0.88295	0.73579
0.78322	0.60248
0.67308	0.48077
0.56177	0.37451
0.45581	0.28488
0.36004	0.21179
0.27718	0.15399
0.20815	0.10955
0.15260	0.07628
0.00189	0.00063

$\theta = \sin^{-1} c$ is such that the hunter, traveling along it with the velocity h, will always intercept the game traveling with the velocity $g = ch$ along any line $\zeta = \zeta_0 > k$. Thus, if the hunter has not encountered the game before becoming blind, that is, at any time $0 \leqslant t \leqslant t_1$, he need merely continue his travel tangent to the trajectory at the instant when he becomes blind, and he will collide with the game even though he is without vision.

It will be noticed that the two-dimensional problem is now solved. The parameters of the problem c and k are supposed to be known. Then, to any $z \leqslant k$ there belongs a $\lambda = 1 - z/k$ and a $\mu = \mu(\lambda)$ found from Table I. (It is not difficult to show that $\mu(\lambda)$ is single-valued because $\lambda(\mu)$ can be shown to be continuous and monotonic.) One may now find the sight angle $\varphi = \sin^{-1}(\mu/\lambda)$ and the flight angle $\theta = \theta(\varphi)$ computed from (4). Again, it is simple to show that, within the domain of interest, $\theta(\varphi)$ exists. Finally, the abscissa belonging to z is found from

$$y = \int_0^{z \leq k} \tan\theta \, dz. \tag{9}$$

For $z > k$, the desired trajectory is

$$y(z) = y(k) + z \tan (\sin^{-1} c). \tag{10}$$

Two-dimensional trajectories for $k = 1$ and $c = 0, \pm 0.2, \pm 0.4$ are shown in Fig. 3. It is immediately evident from this figure (and intuitively appealing as well) that the pursuit curves depend strongly on the velocity ratio c. Not at all obvious, and deserving of special notice is the remarkable property of the solution that the sight angle φ is quite independent of this ratio and depends, in fact, only on z/k, the hunter's z-coordinate measured in terms of his initial sight range.

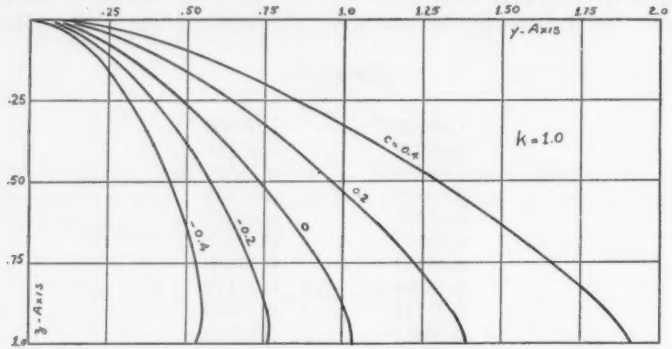

FIG. 3.

We consider briefly the problem where the hunter, rather than colliding with the game, fires a projectile at it from a rifle which is pointed in the direction of the hunter's instantaneous velocity. Measured with respect to fixed coordinates (such as the yz-system) the projectile's velocity p will be greater than the hunter's. Let it be $p = nh$, $n > 1$. Clearly, the kinematical equations of motion of hunter and game remain unchanged. However, the pursuit principle given in (4) has the quantity c replaced by $c' = g/nh = g/p$. The trajectory equation replacing (5) becomes now

$$(1 - 1/n)c + c' = \sin\theta + \frac{d}{dt}\left[\left(\frac{k}{h} - \int_0^t \cos\theta\, dt\right)\frac{\sin\theta - c'}{\sqrt{c'^2 - 2c'\sin\theta + 1}}\right]$$

and, as it should, this equation reduces to (5) for $n = 1$. Unfortunately, the transformations which led to the solution of (5) do not succeed in reducing the above equation to tractable form. In fact, at this time, no way is known of finding the solution to this equation in a simple

manner. One may, of course, examine the case of $n = 1 + \epsilon$, where $0 < \epsilon \ll 1$, by performing perturbations in the neighborhood of the solutions to (5). However, the information to be gained from this approach is limited. For the case where n does not satisfy the above restriction, the equation can, of course, always be solved by numerical methods or by means of analog computers.

THE THREE-DIMENSIONAL PROBLEM

The formulation of the three-dimensional problem differs in few respects from that of the two-dimensional case. The rules regarding the motion of the game are exactly as before, that is, the game travels in a known plane with the constant velocity g along the line $\zeta = \zeta_0$ where ζ_0 is an unknown constant. We call the plane in which the game moves the "plane of action" of the game. As before the hunter travels with the constant speed h, $|h| > |g|$, and his range of vision is $k - z$ for $z \leqslant k$; it remains zero for all $z > k$. The essential new feature is that at $t = 0$, and in general for all $0 \leqslant t < t_1$ the hunter is not in the plane of action of the game. Therefore, the desired trajectory is not a plane curve unless the initial conditions are such that the three-dimensional problem degenerates to the two-dimensional case.

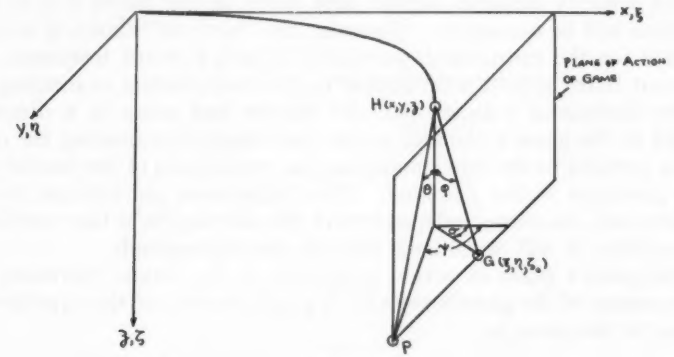

FIG. 4.

A diagram illustrating the geometry of the three-dimensional case, and completely equivalent to Fig. 1 of the two-dimensional problem is shown in Fig. 4. Without loss of generality, we have rotated the xyz-coordinate system about the z-axis until the y-axis is parallel to the plane of action of the game. The position of the hunter is $H(x, y, z)$, that of the game $G(\xi, \eta, \zeta_0)$, and P is the future position common to hunter and game provided the hunter continues tangent to his (curved) trajectory at the instant when he sees the game. As in the two-dimensional case, the angle included between the hunter's line of sight and

the z-axis is called φ, that between the tangent to the hunter's trajectory and the z-axis is θ. However, in order to define uniquely the hunter's directions of sight and flight, two additional angles must be specified.

We have chosen the angles between the projections on the xy-plane of the hunter's lines of sight and flight on the one hand, and the x-axis on the other; these are designated as σ and ψ, respectively. Clearly, reduction to the two-dimensional case occurs when $\sigma = \psi \equiv \pi/2$.

The kinematical equations of motion of the hunter are seen to be

$$\left.\begin{aligned} \dot{x} &= h \sin\theta \cos\psi \\ \dot{y} &= h \sin\theta \sin\psi \\ \dot{z} &= h \cos\theta \end{aligned}\right\}. \tag{11}$$

The best choice of coordinates for describing the motion of the game is not obvious. In the two-dimensional case we proceeded by saying that the game's velocity has no ζ-component and, that the η-component of the relative velocity between hunter and game has the value given in the first of (3). Evidently, the procedure in the three-dimensional case will be analogous; in fact the statement regarding the ζ-component of the game's velocity is the same in both cases. To define completely the relative velocity between hunter and game in the plane $\zeta = \zeta_0$ two equations will be necessary. However, the "obvious" choice of writing relations for the components parallel to x and y is not fortunate. It turns out that the best procedure is to give one relation evaluating the relative horizontal velocity between hunter and game in a direction parallel to the game's plane of action, and another evaluating the component parallel to the line connecting the projections of the hunter and game positions in the xy-plane. These directions are intrinsic to the problem and, therefore, independent of the orientation of the coordinate system; also, it will be noticed, they are not orthogonal.

The game's plane of action is parallel to the y-axis; therefore, the y-component of the game's velocity is g and the first of the equations of motion of the game is

$$g = \dot{y} + \frac{d}{dt}[(k - z) \sin\varphi \sin\sigma]. \tag{12}$$

The projection on the xy-plane of the hunter's velocity is $h \sin\theta$, that of the game is g, and the angle between them is $\pi/2 - \psi$. If g is decomposed into a component parallel to, and one normal to $h \sin\theta$, the relative velocity component between them in the xy-plane may be written as

$$\sqrt{(h \sin\theta - g \sin\psi)^2 + g^2 \cos^2\psi} = h\sqrt{(\sin\theta - c \sin\psi)^2 + c^2 \cos^2\psi}$$

and this velocity must equal the time derivative of the xy-projection of the distance between them or

$$\frac{d}{dt}[(k - z)\sin\varphi] = h\sqrt{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi}. \tag{13}$$

This is the second of the equations for the relative velocity between hunter and game. It remains to formulate equations expressive of the "pursuit principle" and equivalent to (4) in the two-dimensional problem. In the three-dimensional case, conditions on the direction of the tangent to a space curve imply constraints on two partial derivatives. Thus, it is not surprising that we should find two pursuit principles. These are deducible from the three-dimensional velocity space of Fig. 5, and they are

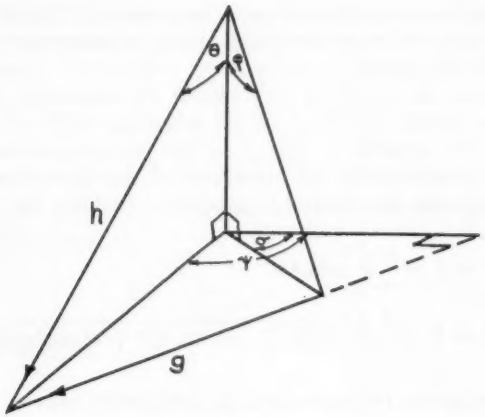

FIG. 5.

$$\left.\begin{aligned} &\tan\varphi = \sqrt{\left(\tan\theta - \frac{c\sin\psi}{\cos\theta}\right)^2 + \frac{c^2\cos^2\psi}{\cos^2\theta}} \\ \text{or}\quad &\sin\varphi = \sqrt{\frac{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi}{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi + \cos^2\theta}} \end{aligned}\right\} \tag{14}$$

and

$$\left.\begin{aligned} &\tan\sigma = \tan\psi - \frac{c}{\sin\theta\cos\psi} \\ \text{or}\quad &\sin\sigma = \frac{\sin\theta\sin\psi - c}{\sqrt{(\sin\theta\sin\psi - c)^2 + \sin^2\theta\cos^2\psi}} \end{aligned}\right\}. \tag{15}$$

For $\psi = \sigma \equiv \pi/2$, (12) and (13) reduce to (3), (14) reduces to (4), and (15) become identities. When substitutions are made similar to those which were required to arrive at (5), the equations of the trajectory are obtained as

$$\frac{d}{dt}\left[\left(\frac{k}{h} - \int_0^t \cos\theta\, dt\right)\sqrt{\frac{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi}{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi + \cos^2\theta}}\right] + \sqrt{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi} = 0 \quad (16)$$

and

$$\frac{d}{dt}\left[\frac{k-z}{h}\sin\varphi\,\frac{\sin\theta\sin\psi - c}{\sqrt{(\sin\theta\sin\psi - c)^2 + \sin^2\theta\cos^2\psi}}\right] + \sin\theta\sin\psi - c = 0. \quad (17)$$

In (17), the equivalents of z and $\sin\varphi$ as given in (11) and the second of (14) are not used, because avoiding these substitutions facilitates the subsequent development.

The problem is now fully formulated by means of (11), (16) and (17) and the initial conditions $x(0) = y(0) = z(0) = 0$, $\theta(0) = \pi/2$, $\sigma(0) = \sigma_0$. The experience gained in the two-dimensional problem is our guide in transforming the equations of the three-dimensional case into a form suitable for further treatment. In (16), let

$$\left.\begin{aligned}\lambda' &= 1 - \frac{h}{k}\int_0^t \cos\theta\, dt \\ \mu' &= 1 - \frac{h}{k}\int_0^t \sqrt{(\sin\theta - c\sin\psi)^2 + c^2\cos^2\psi}\, dt\end{aligned}\right\}. \quad (18)$$

Then, that equation reduces again to the simple form

$$\frac{d\lambda'}{d\mu'} = \sqrt{\left(\frac{\lambda'}{\mu'}\right)^2 - 1} \quad (19)$$

identical with (7). If we define $t = 0$ as that instant at which the projection on the xy-plane of the distance between hunter and game is k and if the hunter is constrained to the plane $z = 0$ for $t < 0$ the initial condition on (19) is $\lambda'(1) = 1$ as in the two-dimensional case. Physical interpretations of λ' and μ' similar to those of λ and μ in the two-dimensional problem show that $k\lambda'$ is the actual distance between hunter and game and $k\mu'$ is the projection of that distance on the xy-plane. Therefore, they define a cone with side $k\lambda'$ and radius of its base circle $k\mu'$. The vertex of this cone coincides with the hunter's position, and the game is located somewhere on the base circle. Consequently, the cone angle is $\varphi = \sin^{-1}(\mu'/\lambda')$ and this is the hunter's sight angle. Clearly,

the $\lambda'\mu'$-correspondence is the same as that between λ and μ because both are identical particular solutions of the same equation. Thus, corresponding pairs of λ' and μ' may be taken from Table I. Moreover, the connection between the hunter's range of sight and his z-coordinate is the same in both problems. Therefore, the relation between z and μ' is the same as that between z and μ in the two-dimensional case.

It remains to solve (17). In that equation we write $(k - z) \sin \varphi = k\mu'$, because $(k - z) \sin \varphi$ is the xy-projection of the distance between hunter and game. Let us now transform (17) by

$$\left.\begin{aligned} m &= \int_0^t (\sin\theta \sin\psi - c)\, dt \\ n &= \int_0^t \sin\theta \cos\psi \, dt \end{aligned}\right\}. \tag{20}$$

The meaning of (20) is that m and n are h-times the y and x-components, respectively, of the projection on the xy-plane, of the distance between hunter and game. Therefore, we also have

$$\left.\begin{aligned} k\mu' &= hm\sqrt{1 + (n/m)^2} \\ \sigma &= \tan^{-1}(m/n) \end{aligned}\right\}. \tag{21}$$

The substitution of (20) and the first of (21) into (17) gives (after we reject the solution $m \equiv 0$)

$$\frac{dn}{dm} = \frac{n}{m}$$

which has the integral

$$m = n \tan\sigma_0. \tag{22}$$

The value of σ_0 is not arbitrary. When information of the game's position, exclusive of ζ_0, is known to the hunter he proceeds in the plane $z = 0$ toward that point at which he would collide with the game if it were moving in the plane $\zeta = 0$, that is, in the same plane as the hunter. The angle between the hunter's direction and the game's plane of action is now $\pi/2 - \psi_0$ where $\psi_0 = \psi(0)$. Then $\sigma(0) = \sigma_0$ is found in accordance with the first of (15) as

$$\sigma_0 = \tan^{-1}[(\sin\psi_0 - c)/\cos\psi_0].$$

Looking in the direction σ_0, the hunter continues in the plane $z = 0$ on the course defined by ψ_0 until the xy-projection of the distance between hunter and game is k. If he does not see the game at this instant, then the game is necessarily at a position in its plane of action for which

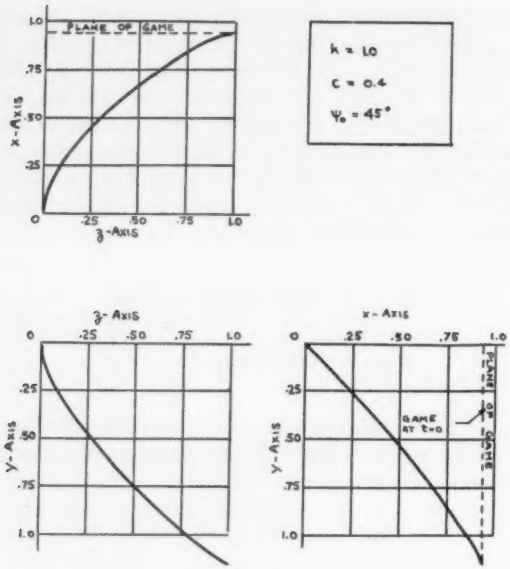
PLANE OF GAME
x-AXIS
ζ-AXIS
k = 1.0
c = 0.4
Ψ₀ = 45°
ζ-AXIS
y-AXIS
x-AXIS
y-AXIS
GAME AT t=0
PLANE OF GAME

FIG. 6.

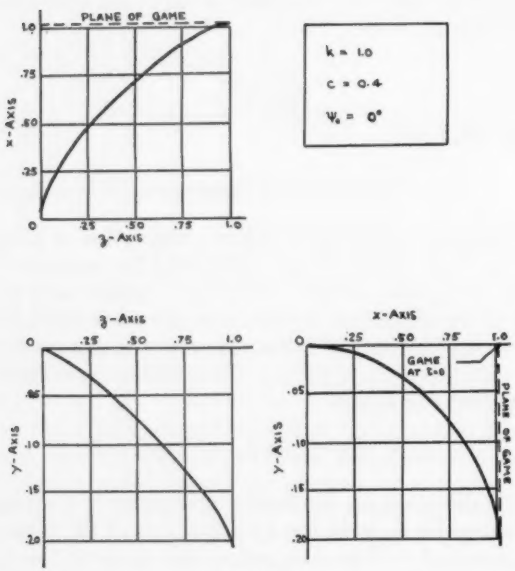
PLANE OF GAME
x-AXIS
ζ-AXIS
k = 1.0
c = 0.4
Ψ₀ = 0°
ζ-AXIS
y-AXIS
x-AXIS
y-AXIS
GAME AT t=0
PLANE OF GAME

FIG. 7.

$\zeta_0 \neq 0$. In this case the hunter engages on the pursuit course defined by our solution as follows:

To every z of the hunter's position there belongs $\lambda' = 1 - z/k$ and a corresponding μ' obtained from Table I. This pair defines the sight angle $\pi \leqslant \varphi \leqslant -\pi$ as $\varphi = \sin^{-1}(\mu'/\lambda')$. But, the first of (15) implies $\theta = \theta(\psi)$; therefore, the first of (14) is of the form

$$\tan \varphi = R(\psi) (= [(\lambda'/\mu')^2 - 1]^{-\frac{1}{2}}) \tag{23}$$

where the latter equality is inherent in the relation between λ' and μ' on the one hand and φ on the other. Since it is not difficult to show that

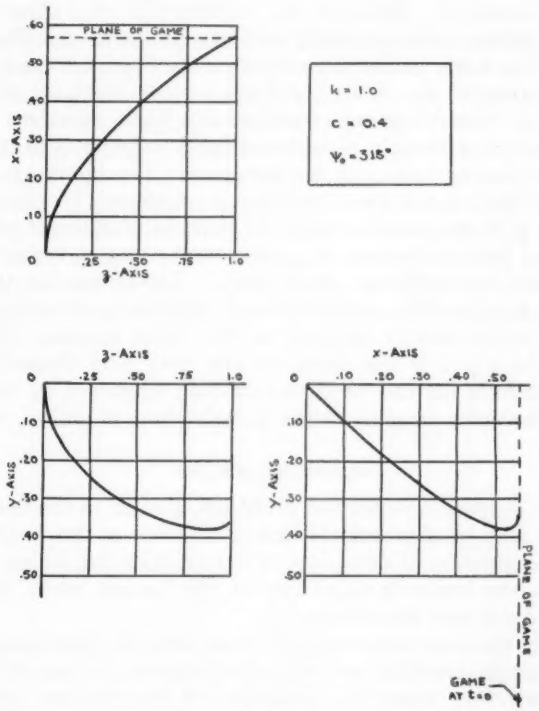

FIG. 8.

$\psi = \psi(\varphi) = \psi(\lambda', \mu')$, that is, the inverse of (23), exists,—the value of ψ associated with any hunter coordinate z is now known. This permits the determination of θ from the first of (15) written in the form

$$\sin \theta = c/(\sin \psi - \tan \sigma_0 \cos \psi). \tag{24}$$

Finally, the x and y-coordinates of the hunter's trajectory, and belonging to a given value of z, are found by means of the quadratures

$$y = \int_0^{z \leq k} \tan\theta \sin\psi \, dz$$
$$x = \int_0^{z \leq k} \tan\theta \cos\psi \, dz. \tag{25}$$

Examples of three-dimensional pursuit curves for $k = 1$, $c = 0.4$, and $\psi_0 = 45°, 0°, 315°$ are shown in Figs. 6, 7, and 8.

We note some interesting properties of the three-dimensional problem. It is a three parameter problem, the three parameters being k, c, and σ_0 (for instance). Although the trajectory is not a plane curve, the problem separates quite naturally into two problems described by (16) and (17). The latter deals with matters in the xy-plane, the former with those orthogonal to it. Both problems become tractable with the introduction of nonorthogonal coordinates. Their solutions prescribe, among other quantities, the directions in and orthogonal to the xy-plane in which the hunter must look for the game. The latter has a solution identical to that of the two-dimensional problem. In particular, the sight angle φ is the same function of z for very different pursuit trajectories and independent of the parameters c and σ_0 which determine the differences between these trajectories. The former has the remarkably simple solution that the direction of sight in the xy-plane σ remains during the entire search constant at the value $\sigma = \sigma_0$. Finally, the trajectory for $z > k$ is the same for the two- and three-dimensional problems because (as can be demonstrated) when $z = k$, the hunter's trajectory becomes tangent to the game's plane of action, and $\theta(k) = \sin^{-1} c$.

CONCLUDING REMARKS

One can propose a variety of problems, similar to the one discussed here, which may be of practical interest. All are to satisfy the criterion that, in the absence of one piece of information regarding the game, tangents to the hunter's trajectory at the instant when the game is discovered must lead to collision.

Probably the most interesting of these from the standpoint of application, although devoid of mathematical interest, is that of finding the hunter's trajectory when the conditions of the problem are those set forth in the introduction, except that the hunter's range of sight is independent of z and simply a number; say, it is equal to k, a constant. Then, the equation of the trajectory of the two-dimensional problem becomes

$$c = \sin\theta + \frac{k}{h}\frac{d}{dt}\left[\frac{\sin\theta - c}{\sqrt{c^2 - 2c\sin\theta + 1}}\right]$$

and transformation similar to those given in (6) lead to a first order differential equation in which the variables can be separated.

When the hunter's range of sight is $(k - z)/\cos\varphi$, $\cos\varphi \neq 0$ — another problem of some practical interest — the equation of the trajectory is

$$c = \sin\theta + \frac{d}{dt}\left[\left(\frac{k}{h} - \int_0^t \cos\theta\, dt\right)\frac{\sin\theta - c}{\cos\theta}\right]$$

and the transformations of (6) lead to the linear equation

$$\lambda\frac{d\mu}{d\lambda} - \mu = A, \quad A = \text{const.}$$

Another of the generalizations which is easily treated is that where the conditions of the problem are those given in the introduction but the game, although confined to its plane of action, travels along a line which has a known inclination to the xy-plane. Thus, while we have treated the case $\partial\xi/\partial\zeta = \partial\eta/\partial\zeta \equiv 0$ we now propose a generalization such that $\partial\xi/\partial\zeta$ and $\partial\eta/\partial\zeta$ are known constants, zero or nonzero, and distinct or not. This generalization introduces an additional parameter into the problem but no mathematical difficulties arise. However, it is not immediately clear how the problem can be generalized to permit, in the game's plane of action, a path of the form $\zeta = \zeta_0 + \zeta_1(\eta)$ where $\zeta_1(\eta)$ is a known, well behaved but not a linear, function of η, and ζ_0 is an unknown constant.

Research Tool for Scatter Propagation Studies.—Although most present microwave communications systems utilize "line of sight" transmission paths, studies during the past few years have shown that useful amounts of microwave energy can be transmitted around the earth's curved surface beyond the horizon. The mechanism which makes this possible is not fully understood, but scattering by atmospheric irregularities appears to play an important role. To study the characteristics of this "scatter propagation," Bell Telephone Laboratories engineers have designed and built one of the world's largest precision microwave antennas at Holmdel, New Jersey.

A. B. Crawford, H. T. Friis, and W. C. Jakes, Jr., of the Laboratories recently reported on the design and performance of this antenna, which is 60 ft. in diameter and accurately steerable. Built of aluminum, the solid surface of this research tool is a paraboloid accurate to about $\frac{3}{16}$ of an inch. The paraboloid alone weighs $5\frac{1}{2}$ tons and it, together with its supporting and steering structure, is designed to withstand 100 mile-per-hour winds.

The antenna is intended for use at frequencies of 460 and 4000 megacycles, but was also tested at 9400 megacycles. Using calibrated pyramidal horns as standards, the gain was found to be 37.0 ± 0.1 db at 460 mc, 54.6 ± 0.2 db at 3890 mc, and 61.1 ± 0.5 db at 9400 mc. Half-power beam widths at these frequencies were 2.45°, 0.3°, and 0.14°, respectively very close to the computed values. Overall performance, not only in gain and beam width but also in cleanness of the radiation patterns and in minor lobe structure, was excellent even at the highest frequency.

With its characteristics now established, this new tool is being employed in research on the nature of scatter propagation, a technique which may find widespread use for communication systems in the future.

Experimental Graphite Reactor.—Research into atomic chain-reaction physics, aimed at producing fissionable materials more efficiently, will be speeded by a new experimental graphite reactor in operation at the Hanford atomic plant.

Dr. A. B. Greninger, Engineering department manager, reported that General Electric, which operates this AEC plant, will use the new $200,000 test unit to evaluate methods of constructing uranium fuel pieces and distributing them within present atomic reactors. It may also help suggest improved designs for any new reactors built in the United States, he said. The unit is called PCTR, short for Physical Constants Test Reactor.

PCTR, though much smaller than the main Hanford reactors, is "critical"; that is, it sustains a chain reaction. Therefore it develops more neutron flux than the sub-critical test reactors previously employed at Hanford, and gives more accurate results. Unlike these, PCTR does not have to be disassembled after each test. It is expected to permit five-fold increase in tests per year with five-fold decrease in cost per test.

The moderator is a graphite block $7 \times 7 \times 7$ ft. in dimensions. A central cavity in this block measures $2 \times 2 \times 3$ ft. Fissionable fuel of various kinds, sizes, and shapes can be inserted into the cavity in various geometrical arrangements.

The underground cell housing the reactor has 2-to-5-ft. concrete walls and roof to shield operating personnel from atomic radiation.

RELIABLE CIRCUITS USING LESS RELIABLE RELAYS

BY

E. F. MOORE[1] AND C. E. SHANNON[1]

Part II[2]

THE CENTRAL PROBLEM

We now attack our main problem—the design of arbitrarily good circuits from crummy relays arbitrarily poor in reliability. It will be shown that circuits with the $h(p)$ characteristic of Fig. 6 can be constructed; $h(p)$ rising from a value less than δ at $p = a$ to a value greater than $1 - \delta$ at $p = c$, for any $\delta > 0$, $0 \leq a < c \leq 1$. We also make estimates of the number of contacts in such circuits in terms of δ, a, and c.

The general design problem will be solved in three stages. These will be called the *opening*, *middle game*, and *end game*. The opening consists of finding a network which, roughly speaking, moves the given a and c over until they straddle the point $p = 1/2$. That is, a network is designed such that $h(a) < 1/2$ and $h(c) > 1/2$. This network can then be thought of as a single contact with more suitably placed a and c (at least, more suitable for the method of design being used).

The *middle game* consists of designing a network that can efficiently separate a and c near the value $1/2$ until a is in the general neighborhood of $1/4$ and c near $3/4$.

The *end game* consists of designing a network that will efficiently move points near $1/4$ and $3/4$ toward the ends of the scale, 0 and 1, respectively.

The solution of the general problem is then found by placing a copy of the first (opening) network for each contact of the second (middle game) network. A copy of this structure is then placed for each contact of the end game network. The total number of contacts used is clearly the product of the numbers used in each of the three subsidiary problems.

This breakdown into three problems is largely for mathematical convenience. It is likely that the most efficient possible design frequently would not consist of this composition of three networks.

In many cases, of course, the first part or even the first two parts of the solution are unnecessary since a and c already are separated by $1/2$ and sufficiently far apart. This indeed will always be true with normal relays. The general case is chiefly of theoretical interest. In the cases of practical interest, where a and c are already close to 0 and 1, it will be

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shown that the circuits proposed for the end game are quite efficient in contacts—they do not contain many more contacts than a provable lower bound for all circuits.

THE OPENING

The first step will be to show that a network can be constructed with an $h(p)$ curve crossing the line $h(p) = 1/2$ in an arbitrary interval and with an average slope in the interval of at least one half. Further, bounds will be obtained for the number of contacts in the network. More precisely, we prove the following result.

Theorem 5

Given a, c with $0 < a < c < 1$, let $b = \frac{a+c}{2}$, let $d = \max(b, 1-b)$, let $\epsilon = \frac{c-a}{4}$, then there exists a network N having fewer than $\left[\frac{\log \epsilon}{\log d}\right]$ contacts such that $h_N(a) \leq \frac{1}{2} - \epsilon$, $h_N(c) \geq \frac{1}{2} + \epsilon$.

Lemma I. There exist two sequences of networks $N_0, N_1, N_2, \cdots$ and $M_0, M_1, M_2, \cdots$ such that for each i:

N_i has i or fewer contacts

M_i has i or fewer contacts

$h_{N_i}(b) < \frac{1}{2} \leq h_{M_i}(b)$

$h_{M_i}(b) - h_{N_i}(b) \leq d^i$

and

either M_i can be obtained by shorting between two of the nodes of N_i

or

N_i can be obtained by opening a contact of M_i.

The networks M_i and N_i in this lemma will be derived from a ladder network of the general form of Fig. 17 (with, however, different numbers of horizontal and vertical elements, depending on the value of b,

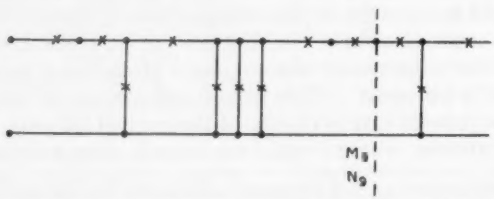

FIG. 17. A ladder network of the sort used in the proof of Theorem 5.

and possibly starting with a vertical instead of a horizontal group). M_i or N_i is obtained by cutting off the network after i contacts (as shown for M_9 or N_9) and shorting or opening, respectively, the wires that are cut. The infinite network may be thought of as one which crosses $h(p) = 1/2$ at exactly $p = b$. The networks M_i and N_i are finite approximations which cross to the left and right of b.

Proof: Let M_0 be a short circuit and N_0 be an open circuit. These satisfy the conditions for the case $i = 0$, since $h_{M_0}(p) = 1$, $h_{N_0}(p) = 0$.

Then assume the above conditions are satisfied for $i - 1$. If M_{i-1} is obtained by shorting between two nodes of N_{i-1}, let M be the network having a single contact added between these nodes. If N_{i-1} is obtained by opening a contact of M_{i-1}, let M be the network obtained by putting another contact in series with this contact. Then M has i or fewer contacts; shorting the added contact causes M to become M_{i-1}, and opening the added contact causes M to become N_{i-1}. Then by (3)

$$h_M(p) = p \cdot h_{M_{i-1}}(p) + (1 - p) \cdot h_{N_{i-1}}(p),$$

hence $h_{N_{i-1}}(p) < h_M(p) < h_{M_{i-1}}(p)$. If $h_M(b) < \frac{1}{2}$, let $N_i = M$, and $M_i = M_{i-1}$. Then

$$\begin{aligned} h_{M_i}(b) - h_{N_i}(b) &= h_{M_{i-1}}(b) - h_M(b) \\ &= h_{M_{i-1}}(b) - [b\, h_{M_{i-1}}(b) + (1 - b)\, h_{N_{i-1}}(b)] \\ &= (1 - b)\,(h_{M_{i-1}}(b) - h_{N_{i-1}}(b)) \\ &\leq (1 - b)d^{i-1} \leq d^i. \end{aligned}$$

And if $h_M(b) \geq \frac{1}{2}$, let $M_i = M$ and $N_i = N_{i-1}$. Then similarly $h_{M_i}(b) - h_{N_i}(b) \leq bd^{i-1} \leq d^i$. This completes the proof of the lemma.

Lemma II. $h'(p) \geq \frac{3}{4}$ for all p such that $\frac{1}{2} - \epsilon \leq h(p) \leq \frac{1}{2} + \epsilon$.

Since $\epsilon = \dfrac{c - a}{4} \leq \frac{1}{4}$, we have $\frac{1}{4} \leq h(p) \leq \frac{3}{4}$, hence $h'(p) \geq \dfrac{(h - 1)h}{(p - 1)p} \geq \dfrac{\frac{3}{16}}{\frac{1}{4}} = \frac{3}{4}$.

To prove the original theorem, let $i = \left[\dfrac{\log \epsilon}{\log d}\right]$, then $d^i \leq \epsilon$, hence

$$h_{N_i}(b) < \tfrac{1}{2} \leq h_{M_i}(b) \leq h_{N_i}(b) + \epsilon.$$

Let N be whichever of N_i or M_i will satisfy $|h_N(b) - \frac{1}{2}| \leq \dfrac{\epsilon}{2}$, since one of the two must satisfy it.

Then, without loss of generality, it suffices to show only that $h_N(c) \geq \frac{1}{2} + \epsilon$.

Assume the contrary, that $h_N(c) < \frac{1}{2} + \epsilon$. Then since h_N is monotone, we have $\frac{1}{2} - \epsilon \leq h(p) \leq \frac{1}{2} + \epsilon$ for all p between b and c. Hence by Lemma II. $h'(p) \geq \frac{3}{4}$ in this interval.

Hence

$$h(c) = h(b) = \int_b^c h'(p)\,dp \geq \frac{1}{2} - \frac{\epsilon}{2} + \frac{3}{4}\cdot(c - b)$$

$$\geq \frac{1}{2} - \frac{\epsilon}{2} + \frac{3\epsilon}{2} \geq \frac{1}{2} + \epsilon$$

which contradicts the assumption.

THE MIDDLE GAME

The second phase of our approach consists of finding networks which move probabilities just below and just above 1/2 to values beyond 1/4 and 3/4, respectively. Thus we seek networks for which

$$h(1/2 - \epsilon) \leq 1/4$$

$$h(1/2 + \epsilon) \geq 3/4,$$

and wish to estimate the number of elements used in terms of ϵ.

Networks satisfying our requirements may be obtained by composing a self-dual network with itself a sufficient number of times. For example, the three-by-three hammock network, Fig. 16, may be used. It is not difficult to show that the $h(p)$ function for this network, as shown in Fig. 18, lies below a straight line of slope 3/2 passing through

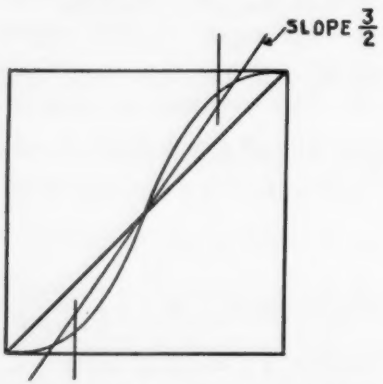

FIG. 18. The function associated with the three-by-three hammock network.

the point 1/2, 1/2 in the interval $1/4 \leq p < 1/2$, and lies above this straight line in the interval $1/2 < p \leq 3/4$. This means that composing the hammock network with itself a sufficient number of times to move a point $1/2 - \epsilon$ out to 1/4 will require fewer compositions than if the staircase construction moved along this straight line. Now, if we move along the straight line starting at $1/2 - \epsilon$, it is evident that after

s compositions (or staircase steps) the point will be moved to $1/2 - \epsilon \times \left(\frac{3}{2}\right)^s$. If we wish to reach $1/4$, then s compositions will suffice if s is the smallest integer satisfying

$$\frac{1}{4\epsilon} = \left(\frac{3}{2}\right)^s,$$

and this same number will surely suffice with the hammock network since its effect in the staircase construction is stronger. The number of contacts for an s-fold composition of this hammock network with itself is

$$N_2 = 9^s \leq 9\left(\frac{1}{4\epsilon}\right)^{\frac{\log 9}{\log 3/2}}$$

$$\leq 9\left(\frac{1}{4\epsilon}\right)^{5.41}.$$

The same considerations apply, of course, to moving the point $1/2 + \epsilon$ to $p = 3/4$ and the same network, being self-dual, performs this function.

Since our ϵ resulting from the opening part of the strategy was $\frac{c-a}{4}$, the middle game network requires at most $9\left(\frac{1}{c-a}\right)^{5.41}$ contacts. The combined network requires not more than $9\left[\frac{\log \frac{c-a}{4}}{\log d}\right] \times \left(\frac{1}{c-a}\right)^{5.41}$ contacts.

THE END GAME

Our third "end game" network must take probabilities at $1/4$ and $3/4$ or better and move them to points near 0 and 1, respectively. This type network will also be found by composing a self-dual network with itself, in particular the three-by-three hammock or the five-element bridge. It will first be convenient to carry out some calculations to determine the rate at which it is possible to approach 0 and 1 in such cases.

It is sometimes possible to obtain upper or lower bounds on a particular $h(p)$ function in the form of a power of p. For example

$$h(p) \leq ap^r. \tag{16}$$

If $h(p)$ is composed with itself we obtain, using the fact that $h(p)$ is monotone:

$$h^{(2)}(p) \leq a(ap^r)^r = a^{1+r}p^{r^2}$$
$$h^{(3)}(p) \leq a^{1+r}(ap^r)^{r^2} = a^{1+r+r^2}p^{r^3}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$h^{(s)}(p) \leq a^{1+r+r^2+\cdots+r^{s-1}}p^{r^s}$$
$$= a\frac{r^{s-1}}{r-1}p^{r^s}$$
$$= \frac{(a^{\frac{1}{r-1}})^{r^s}p^{r^s}}{a^{\frac{1}{r-1}}}. \tag{17}$$

Now suppose that the original network contains n_1 contacts. When composed with itself s times, the resulting network will require $n = n_1^s$ contacts. Eliminating s from (17), we obtain

$$h^{(s)}(p) \leq \frac{(a^{\frac{1}{r-1}})^{n^{\frac{\log r}{\log n_1}}}p^{n^{\frac{\log r}{\log n_1}}}}{a^{\frac{1}{r-1}}}.$$

If the direction of the inequality (16) were reversed the result would also be reversed. Furthermore, similar results hold with p replaced by $1 - p$.

The three-by-three hammock network, Fig. 16, has the $h(p)$ function:

$$h(p) = 8p^3 - 6p^4 - 6p^5 + 12p^7 - 9p^8 + 2p^9$$
$$= 8p^3 - p^4[6 + 6p - 12p^3 + 9p^4 - 2p^5]$$
$$= 8p^3 - p^4[6(1 - p^3) + 6p(1 - p^2) + 7p^4 + 2p^4(1 - p)].$$

Since the bracketed expression is clearly non-negative,

$$h(p) \leq 8p^3 \quad \text{for} \quad 0 \leq p \leq 1.$$

Hence for the s'th composition, by (17):

$$h^{(s)}(p) \leq \frac{1}{\sqrt{8}}(\sqrt{8}\,p)^{3^s}.$$

Since each composition multiplies the number of contacts by 9, the total number is $N = 9^s$, $\sqrt{N} = 3^s$.

$$h^{(s)}(p) \leq \frac{1}{\sqrt{8}}(\sqrt{8}\,p)^{\sqrt{N}}. \tag{18}$$

$$N \leq \left(\frac{\log \sqrt{8}\,h^{(s)}(p)}{\log \sqrt{8}\,p}\right)^2. \tag{19}$$

It may be noted that this gives a significant result only when $p \leq \frac{1}{\sqrt{8}} \doteq 0.353$. For larger values of p the upper bound on h exceeds 1.

It should also be pointed out that in this process N must jump by multiples of nine. If the right member of (19) is given, the number N of contacts required may be up to nine times that defined by the inequality, but this amount will surely suffice.

The dual situation gives an inequality for N:

$$N \leq \left[\frac{\log \sqrt{8}\,(1-h)^{(*)}(1-p)}{\log \sqrt{8}\,(1-p)} \right]^2. \tag{20}$$

Returning now to the end-game problem, we wish to improve a probability $p \leq 1/4$ to $h(p) \leq \delta$, say. This requires by (19) not more than $9\left(\frac{\log \sqrt{8}\,\delta}{\log \sqrt{2}}\right)^2$ contacts. Since the hammock network is self-dual, similar considerations apply to improving the probability $3/4$ to $1-\delta$. The same network will effect this improvement.

We may summarize the results obtained by the composition of these three networks in the following manner:

Theorem 6

Given $\delta > 0$, $0 < a < c < 1$, let $d = \max \frac{a+c}{2}, 1 - \frac{a+c}{2}$.

There exists a network such that

$$h(a) < \delta$$
$$h(c) > 1 - \delta$$

using not more than N contacts, where

$$N = 81 \left[\frac{\log \frac{c-a}{4}}{\log d} \right] \left(\frac{1}{c-a} \right)^{\frac{\log 9}{\log 3/2}} \left(\frac{\log \sqrt{8}\,\delta}{\log \sqrt{8}} \right)^2.$$

This result shows that the equivalent of an arbitrarily reliable relay may be made from a sufficient number of arbitrarily poor ones, and places a bound on the number of them which are necessary. The bound is undoubtedly quite loose. In particular the factor 81, which was introduced because things might not "come out even," could probably be reduced a great deal by a more refined analysis. For particular values of c, a and δ, this factor will be one, and for cases where the factor is large, the substitution of other networks for the three-by-three ham-

mock will often reduce it to a number not far from one, without greatly changing other parts of the bound.

CONTACT BOUNDS WHEN THE RELAYS ARE ALREADY QUITE GOOD

If the relays are initially quite reliable, the first two stages of the above construction can be omitted, the a and c being less than $1/4$ and greater than $3/4$ at the start. This of course is the usual practical case. When this is true, and we wish to improve a down to δ_1 and c up to $1 - \delta_2$, not more than the maximum of:

$$9\left(\frac{\log \sqrt{8}\,\delta_1}{\log \sqrt{8}\,a}\right)^2, \quad 9\left(\frac{\log \sqrt{8}\,\delta_2}{\log \sqrt{8}\,(1-c)}\right)^2$$

contacts are required, and if the numbers work out right, the factor 9 may be replaced by a factor as low as one.

In the appendix we develop some similar bounds using general hammock networks. These bounds are somewhat stronger, both in eliminating the factor 9 and in replacing the coefficient $\sqrt{8}$ by a smaller one.

We will now develop a companion inequality to this giving a *lower* bound for the number of contacts required for a given improvement in reliability.

Theorem 7

Suppose $0 < a < c < 1$ and N is a two-terminal network with $h(p)$ satisfying

$$h(a) \leq \delta_1$$
$$h(c) \geq 1 - \delta_2.$$

Then the number of contacts, n, in the network satisfies

$$n \geq \frac{\log \delta_1}{\log a} \cdot \frac{\log \delta_2}{\log (1-c)}.$$

For example, if contacts made errors one time in ten, both when they should be open and when they should be closed, we would have $a = 1 - c = 10^{-1}$. If a network is desired making but one error in 10^6 operations, it will require by this theorem at least

$$\frac{\log 10^{-6}}{\log 10^{-1}} \cdot \frac{\log 10^{-6}}{\log 10^{-1}} = 36 \text{ contacts.}$$

To prove this theorem, let the network N have length l and width w. There is then a path through l contacts from one terminal to the other. The probability that this path will be closed when it should be open is a^l.

If this should occur, the network of course will be closed when it should be open. Hence

$$\delta_1 \geq a^l$$

and

$$\log \delta_1 \geq l \log a.$$

Dividing by the necessarily negative $\log a$ reverses the inequality:

$$\frac{\log \delta_1}{\log a} \leq l.$$

The dual argument concerning cut-sets with w elements goes through in a similar fashion to give

$$\frac{\log \delta_2}{\log (1 - c)} \leq w.$$

Since all terms are positive, we may multiply these inequalities and obtain

$$\frac{\log \delta_1}{\log a} \cdot \frac{\log \delta_2}{\log (1 - c)} \leq lw \leq n,$$

using the inequality proved in Theorem 3.

To summarize, the number of contacts n required to improve the probability of error on make from a to δ_1 and on break from $\log (1 - c)$ to δ_2 is something like

$$\frac{\log \delta_1}{\log a} \cdot \frac{\log \delta_2}{\log (1 - c)}.$$

It is never less than this, and for an infinite sequence of increasing values of n only a little greater, as shown for the hammock networks in the Appendix.

COMPARISON WITH VON NEUMANN'S ELEMENTS

As a numerical example, we may consider a case similar to one used by von Neumann, in which he assumes Sheffer stroke organs whose probability of error is 1/200 and wishes to construct from them a Sheffer stroke organ with a probability of error about 10^{-20}. He finds that his circuits then require something like 60,000 elements for each desired reliable organ. It also turns out that the number 60,000 is quite insensitive to the final reliability 10^{-20}, varying only from 32,000 to 69,000 when the final reliability varies from 10^{-17} to 10^{-23}.

As a corresponding problem we may consider relays with initial probabilities of error $a = 1 - c = 1/200$, and ask for circuits which improve this reliability to figures like 10^{-20}. Since our initial probabilities are relatively good, we need only use the end-game type of analysis.

Let us first consider using the three-by-three hammock networks in composition. If one stage is used, a redundancy of 9 to 1 is involved and the final probability of error is less than $8\left(\frac{1}{200}\right)^3 = 10^{-6}$. If two stages are used the redundancy is 81 to 1, and the final probability of error is less than $8^{-1}\left(\sqrt{8}\cdot\frac{1}{200}\right)^9 = 8\cdot 10^{-18}$. Thus with this redundancy of 81 to 1 we are in the general range desired, although still a bit short. Another composition of the three-by-three hammock gives a redundancy of 729 to 1 and the final error probability is less than $4\cdot 10^{-51}$!

By using larger hammock networks (see the Appendix for bounds on the error probability), we can hit closer to our mark. Thus, with a ten-by-ten hammock, the error will be less than $2\cdot 10^{-19}$ with a redundancy of 100 to 1, while with the eleven-by-eleven the figure is $2.2\cdot 10^{-21}$ with redundancy 121 to 1.

In general, in the range where the Sheffer stroke organs require redundancies of 50,000 up to 70,000, crummy relays require redundancies of only 80 to 120, a rather remarkable reduction.

The lower bound for the number of contacts, obtained in Theorem 7, may now be applied to show that the redundancies here could not be improved very much. In fact, in order to obtain a final probability $8\cdot 10^{-18}$, at least 55 contacts are necessary (while 81 were used). To obtain $2\cdot 10^{-19}$ at least 66 are necessary (100 were used), while for $2.2\cdot 10^{-21}$ at least 81 are necessary (121 were used). In all these cases the lower bound is very nearly two-thirds of the actual number.

It is interesting to speculate on why the relays should require so much less redundancy than appears to be necessary for the Sheffer stroke type of synthesis. (It is not certain,[8] of course, that the type of error control used by von Neumann approximates the most efficient use of these elements.) One difference between the two types of component, which may account for this difference, is the following. In both types of machine two processes are involved: first the duplication of variables, that is, obtaining copies of the same variable for use in various parts of the computation, and second, forming logical combinations and functions of several variables. In the case of crummy relays, the errors occur in the duplication of variables. The different contacts on a relay are not in exact correspondence with the coil but are subject to statistical error. However, logical combinations are formed without error; a series combination is an absolutely correct "and" circuit and a parallel combination is always correct as an "or" circuit.

In the case of neuron-type components, the situation is reversed. It is possible to obtain any number of duplications of a given variable

[8] In fact, it is not even certain that the methods used in this paper give close to the best possible reduction of errors for relay circuits. Perhaps it is more efficient to redesign the circuit as a whole to get redundancy than to replace each relay by an individual circuit.

by merely branching the line on which it appears. These are all assumed to be identical. However, when a logical combination is made of two or more variables in a Sheffer stroke or majority organ, statistical errors occur. Now, in each of the two machines, the statistically unreliable part must be checked by a logical operation involving something akin to vote-taking. "Sed quis custodiet ipsos custodies?" In the relay case, the custodians are beyond reproach; in the case of neuron-like organs, the custodians must indeed be carefully controlled by further custodians. This fundamental difference may be responsible for the difference in the redundancies and possibly also for the fact that a certain reliability is necessary in the Sheffer organs in order to control errors at all, a situation that is not present in the relay case.

It is interesting, in this comparison, that for sufficiently great improvements in reliability the von Neumann circuits will require fewer elements than the ones we have described. If we take as before $\delta_1 = \delta_2$ and $a = 1 - c = 0.005$, he has approximately $\left($taking the logarithm of his Eq. 27, identifying $p = a$ and ignoring the higher order term $\log \frac{6.4}{\sqrt{n}}\right)$

$$n \approx - 3500 \log \delta_1.$$

With our circuits, we have

$$n \approx \tfrac{1}{4} (\log \delta_1)^2.$$

These curves cross at $\delta_1 \approx 10^{-14,000}$! If improvements in reliability greater than this were desired, the multiplexing circuits would require fewer elements.

However, the effect of a Sheffer stroke organ could be obtained by using the circuit of Fig. 19, in which two relays, each having one back

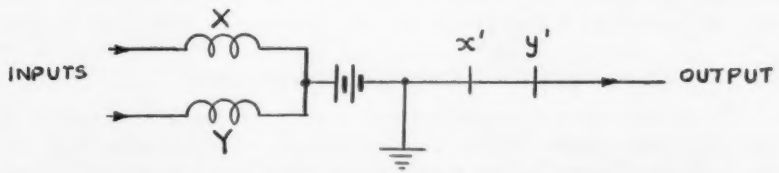

FIG. 19. Method of making a Sheffer stroke element from two relays.

contact, labelled x' and y', respectively, are connected to act as such an organ. This circuit as a Sheffer stroke has a reliability only slightly less than its individual elements and may be used in the type of circuit proposed by von Neumann. Hence, for the extremely high reliabilities

where the von Neumann circuits have the advantage, it is possible to change over to the first power increase with $\log \delta_1$ using only slightly greater redundancy than the Sheffer stroke elements.

The converse method, attempting to use the relay type correcting circuit for the Sheffer stroke organs, does not appear to be possible. There seems to be no way to make anything which acts quite like a relay from Sheffer stroke organs.

RELAYS WITH UNCERTAIN OPERATE TIMES

The function $h(p)$ associated with a network of contacts can be interpreted in a way somewhat different from that employed above. Suppose that each of the contacts in the network is on a different relay and that these relays are uncertain in their operate times. Let $\varphi(t)$ be the cumulative distribution function for one of the relays; if the relay is energized at $t = 0$, $\varphi(t)$ is the probability that its contact will be closed at time t. The same $\varphi(t)$ is assumed to apply to all the relays and they are assumed statistically independent. Then *the cumulative distribution for the two-terminal network is given by* $h[\varphi(t)]$. The reason for this is that at any time t_1 each of the contacts may be thought of as a contact on a crummy relay whose probability of closure is $p = \varphi(t_1)$. Hence, the probability of the network as a whole being closed at t_1 is $h(p) = h[\varphi(t_1)]$.

The same statements hold even if the relays are both uncertain in time and crummy. In this case, $\varphi(t)$ does not range from zero to one but from the c to a of Fig. 1, when t goes from zero to $+\infty$.

The results we proved concerning the sharpening of the $h(p)$ curve show in this interpretation that relays of uncertain operate time can be used to synthesize circuits of precise timing. The fact that $h(p)$ crosses the diagonal at most once shows that, in a certain sense, the timing of any two-terminal circuit is less uncertain than its component contacts. In a general way this interpretation indicates that replacing individual contacts by these two-terminal networks should tend to improve timing margins and reduce the probability of errors due to races and the like.

DESIGN OF RELIABLE CIRCUITS IN THE LARGE

Up to now we have been concerned with the problem of designing circuits which would act like a single reliable relay contact. It is not immediately clear that we may replace the relay contacts of a large circuit by the reliable circuits we have developed and necessarily expect the large circuit to behave reliably. The difficulty is that replacing a single contact by a reliable network introduces the possibility of certain race conditions which might conceivably cause errors of a kind not present in the original network. For example, one of our reliable networks might open and close rapidly several times in the transition of its relays

from the energized to the non-energized state. If this circuit operated a pulse-counting circuit which could correctly count such a rapid sequence of pulses, it is evident that errors could be introduced. We have not been able to justify the use of these reliable circuits in all cases for just such reasons as these. There are, however, many types of circuit in which such effects cannot occur and there are reasons for believing that even when they can, they will usually not be troublesome. Some possible cases and justifications are as follows.

1. Forward-acting circuit. By this we mean relay circuits in which the relays may be organized in levels. A relay in level n, say, is controlled by a contact network on inputs and relays of levels less than n. This type of organization implies that there are no memory or feedback effects and that race conditions are of no significance for final positions of the relays. It is easy to see that if such a circuit contains N contacts and each of these is replaced by a reliable network with probability less than P of incorrect operation, then the network as a whole will have probability less than NP of failure. This overbound adds the failures of the individual contacts disjunctively and hence pessimistically.

2. "Synchronous" relay circuits. By this we mean circuits operating in a quantized time system and acting somewhat like the neuron models of McCulloch and Pitts[4] or like IBM "selectors." More precisely, contacts can open or close only at integer multiple of the unit T of time, and if at time nT a relay is energized (or not energized), at time $(n + 1)T$ the contacts will have probability a (or c) of being closed. Circuits constructed of such components can be made reliable by techniques we have described, even if memory and feedback are involved. The probability of an error in any calculation performed by such a circuit is, by a slight extension of the previous argument, less than PND, where TD is the duration of the calculation, and P and N are as before.

3. In many ordinary relay circuits the techniques we have described will lead to reliable relay circuits even though feedback and memory are involved. This is because, as we have seen, reliable contact networks tend to sharpen the effective operate time. As the contact network is made more and more reliable it tends to act more and more like a relay with a very definite operate time. If any extra opening and closing occurs, it is with high probability confined to an extremely short time and would not be expected to cause malfunctioning of the circuit. Thus, it is very plausible that even in cases of this sort the techniques we suggest will not, in fact, lead to trouble, particularly if a great deal of improvement in reliability (and hence great sharpening of operate and release times) is involved.

[4] W. S. McCULLOCH AND W. PITTS, *Bull. Math. Biophysics*, Vol. 5, p. 115 (1942).

APPENDIX

Upper Bounds for the Error Probabilities with l-by-w Hammock Networks

In our standard design method we made use of the three-by-three hammock network, composed with itself many times if necessary, to give increased reliability. This led to easily computed upper bounds for the error probabilities. It is plausible, however, that a more efficient procedure might be to use a larger type of hammock network without the composition.

In this Appendix it will be shown that with a, c, δ_1, δ_2 as before, in a hammock network of length l and width w we have the inequalities

$$\delta_1 \leq \left(\frac{1 - \sqrt{1 - 16a^2}}{4a} \right)^{l-1} wa$$

and

$$\delta_2 \leq \left(\frac{1 - \sqrt{1 - 16(1-c)^2}}{4(1-c)} \right)^{w-1} l(1-c).$$

For a and $1 - c$ small, the right members of these inequalities are approximately $(2a)^{l-1}\, wa$ and $[2(1-c)]^{w-1}\, l(1-c)$.

To prove these inequalities, first consider the infinite network shown in Fig. 20. All sloping lines here represent contacts with probability

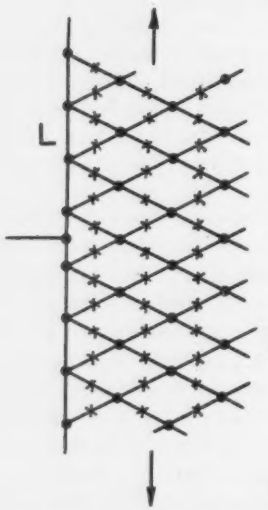

FIG. 20. The infinite hammock network.

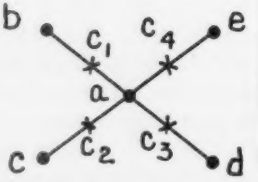

FIG. 21. One node of the infinite hammock network.

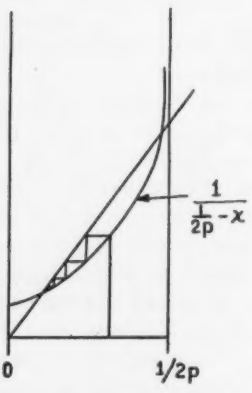

FIG. 22. The iterative procedure used to improve the bound.

p of being closed. The network is assumed to continue indefinitely at the top, bottom and to the right. The vertical line L at the left is a common connection. Let P_1 be the probability of one of the junction points at distance 1 from L being connected to the line L. Similarly P_2 is the probability for those at distance 2, and P_n for those at distance n. We wish to derive an upper bound for P_n.

In Fig. 21, the node a at distance n will be closed only if node b is closed to L and contact c_1 is closed, or node c is closed to L and contact c_2 is closed, or node d is closed to L and contact c_3 is closed, or node e is closed to L and contact c_4 is closed. An upper bound for the probability for the first of these four events is pP_{n-1}. This also applies to the second event. The last two have upper bounds pP_{n+1}. If we add these (as though they were disjunctive) we have again increased our estimate of the probability P_n (since more than one of these can occur at once). Thus we can say

$$P_n < 2pP_{n-1} + 2pP_{n+1}. \tag{21}$$

Now it is evident that $P_{n+1} \leq P_n$. Using this in the above, we obtain

$$P_n < 2pP_{n-1} + 2pP_n$$

$$P_n < \frac{2p}{1-2p} P_{n-1}. \tag{22}$$

This relation, carried back to $P_0 = 1$, gives

$$P_n \leq \left(\frac{2p}{1-2p}\right)^n.$$

However, this can be improved by a curious feedback process, using the relation (22) back in (21) as a better estimate of P_{n+1}. Thus, from (22),

$P_{n+1} \leq \dfrac{2p}{1-2p} P_n$ and using this in (21)

$$P_n < 2pP_{n-1} + \frac{4p^2}{1-2p} P_n.$$

$$P_n < \frac{2p}{1 - \left(\dfrac{4p^2}{1-2p}\right)} P_{n-1} = \frac{2p(1-2p)}{1-2p-4p^2} P_{n-1}.$$

This again can be used to improve the estimate. To find the result of infinite application of this process, notice that at stage j we have a relation

$$P_{n+1} < \alpha_j P_n.$$

Using this in (21) we have

$$P_n < 2pP_{n-1} + 2p\,\alpha_j P_n$$

and hence

$$P_n < \frac{2p}{1 - 2p\alpha_j} P_{n-1}$$

$$\alpha_{j+1} = \frac{2p}{1 - 2p\alpha_j}.$$

This hyperbolic relation between α_{j+1} and α_j is plotted in Fig. 22. If $p < 1/4$, the hyperbola will intersect the straight line $\alpha_{j+1} = \alpha_j$ in two places, namely the roots of

$$2p\alpha^2 - \alpha + 2p = 0$$

$$\alpha = \frac{1 \pm \sqrt{1 - 16p^2}}{4p}.$$

The lower intersection, corresponding to the root $\dfrac{1 - \sqrt{1 - 16p^2}}{4p}$, is a stable point of the iterative process. Starting with any α_0 lying between zero and one, α_n will approach as a limit this root by the staircase process indicated in Fig. 22. It follows readily that

$$P_n \leq \left(\frac{1 - \sqrt{1 - 16p^2}}{4p}\right) P_{n-1}$$

and from this that

$$P_n \leq \left(\frac{1 - \sqrt{1 - 16p^2}}{4p}\right)^n. \tag{23}$$

Now consider the l-by-w hammock network. This can be thought of as made up of the two parts shown in Fig. 23. The probability of

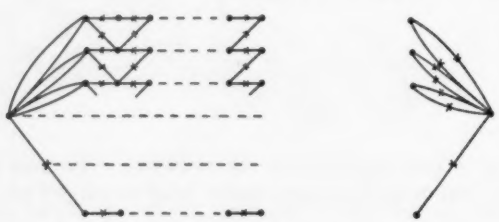

FIG. 23. An l-by-w hammock network is made up of the two parts shown above.

closure for this network is bounded by the inequality

$$h(p) \leq P^{(1)}(2p - p^2) + P^{(2)}(2p - p^2) + \cdots.$$

The last term will be of the form $P^{(s)}(2p - p^2)$ or $P^{(s)}p$, according as w is even or odd. In this expression, $P(j)$ is the probability of the left half of the network being closed to its jth node from the top on the right. The quantities $2p - p^2$ are the probabilities for the parallel connection of two contacts being closed. The terms of the sum therefore correspond to the different ways the network could be closed. Since we have added these, we have an upper bound for the probability.

It is clear that each $P^{(i)} \leq P_{l-1}$ since we may obtain the network of Fig. 20 from the left part of Fig. 23 by adding contacts which will certainly only increase the probability of closure. Consequently, from (23) $P^{(i)} < \left(\frac{1 - \sqrt{1 - 16p^2}}{4p}\right)^{l-1}$. Using this and also strengthening the inequality by deleting the negative p^2 terms, we arrive at the desired result

$$\delta_1 < \left(\frac{1 - \sqrt{1 - 16a^2}}{4a}\right)^{l-1} wa.$$

The dual of a hammock with dimensions l and w is a hammock with dimensions w and l. By duality we obtain

$$\delta_2 < \left(\frac{1 - \sqrt{1 - 16(1 - c)^2}}{4(1 - c)}\right)^{w-1} l(1 - c).$$

It may be noted that for a and $1 - c$ small, these upper bounds become approximately $\frac{w}{2}(2a)^l$ and $\frac{l}{2}[2(1 - c)]^w$. We conjecture, but have not been able to prove, that for all a and c

$$\delta_1 < \frac{w}{2}(2a)^l$$

$$\delta_2 < \frac{l}{2}[2(1 - c)]^w.$$

Helicopter TV Stations.—Helicopters—the intrepid "whirly birds" of numerous military and civilian rescue missions—now may become self-contained flying television stations during major naval operations. Developed at Philco Corporation's Government & Industrial Division for the Navy Department's Bureau of Ships, a new airborne TV system is said to provide excellent operation over line-of-sight distances of more than fifty miles. Systems of this type could be used by the Navy for control of amphibious landings.

The system, an ultra-high frequency FM radio television link, provides for transmission, reception and display of TV information from the air to surface ships, or ground installations. During recent tests with the fleet, the Navy reported that excellent pictures were obtained of all phases of ship-to-shore movement, even before sunrise.

Philco, which has been experimenting with different systems of airborne TV transmission, has just completed additional low-altitude tests over the Philadelphia area. The tests were conducted in the UHF region of 894 megacycles and were from altitudes between 500 and 2500 feet. Philco recently announced that it had developed an airborne TV system for use in jet aircraft reconnaissance by the U. S. Air Force. That system may be used to transmit a TV picture from the stratosphere.

The airborne equipment for the new Navy TV system includes a frequency modulated transmitter of 100 watts effective radiated power, image orthicon camera, coder (timing source and generator) and a high-gain antenna which is dropped below the belly of the helicopter after it is airborne.

Ground installation equipment includes an FM receiver, cathode ray tube indicator and high-gain antenna.

Ultra-Pure Silicon.—Commercially-available silicon, even of the highest purity, contains significant amounts of impurities which must be removed if the material is to be used for semiconductor devices. Of these impurities, the most difficult to remove is boron.

H. C. Theuerer of Bell Telephone Laboratories has found that boron can be removed from molten silicon by reaction with water vapor. This reaction oxidizes the boron, and the oxidation products evaporate.

To carry out the reaction, a liquid silicon zone supported only by surface tension is caused to traverse a vertical silicon rod around which flows a mixture of hydrogen and water vapor. This technique prevents contamination from crucibles and provides a large interface between the silicon and the atmosphere. Under these conditions, removal of boron is very effective, increasing both with time and with water vapor concentration.

This method, used together with the zone refining technique developed at Bell Telephone Laboratories, makes possible the production of silicon concontaining boron at a concentration below one part in ten billion and having a resistivity greater than 3000 ohm-cm. Improved transistors and other semiconductor devices may result from this ultra-pure material.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

PROPAGATION OF SOUND IN MONATOMIC GASES

Measurements of the speed and attenuation of 11-megacycle per second sound waves in monatomic gases were recently completed by the National Bureau of Standards.[1] Data obtained from five gases—helium, neon, argon, krypton, and zenon—have provided new information on the behavior of gases under non-equilibrium conditions, permitting an intercomparison of propagation theories based on "classical" equilibrium assumptions and those based on modern kinetic concepts. Supported in part by the Office of Naval Research, the investigation was conducted by M. Greenspan of the NBS sound laboratory. A double-crystal interferometer in which a number of design improvements have been made was the principal instrument used.

While the immediate purpose of these measurements is to assist in the verification of theories of non-uniform gases, the results have a bearing on upper-atmosphere flight, the propagation of shock waves, and problems of high-speed gas dynamics generally. For example, aerodynamic research has shown that the motion of an aeroplane or missile through the air is decisively affected by conditions in the so-called boundary layer of air immediately surrounding the moving object. This layer is characterized by the occurrence of large changes in pressure and temperature within very short distances, in contrast to the uniformity assumed in classical theories. Thus the classical assumption has proved inadequate for calculating the viscosity of gases surrounding even relatively slowly moving objects. Considerable effort has therefore been made to develop mathematical theories that take the effect of departure from equilibrium into account. The study of sound propagation provides one of the more direct experimental checks on these recent theories.

TRANSLATIONAL DISPERSION

Under ordinary atmospheric conditions and at audio frequencies, there is no detectable dispersion of sound; i.e., all these frequencies travel with the same speed. Otherwise, for example, the sounds from a flute and a bass drum, though emitted simultaneously, would not reach the back of the concert hall at the same time. When the frequency is raised sufficiently, however, or if the pressure is correspondingly reduced, an effect known as translational dispersion occurs.

[1] For further technical details, see "Propagation of Sound in Five Monatomic Gases," by M. Greenspan, *J. Acoust. Soc. Am.*, Vol. 28 (July 1956).

Though long expected on the basis of theory, this effect was first detected experimentally by the NBS sound laboratory in 1949.[2]

Translational dispersion can be traced to the nonuniform conditions created by the passage of a sound wave. In one region of the gas, for example, the molecules will be moving with an average speed somewhat in excess of that of the molecules a little farther on in the direction of propagation. Ordinarily the molecular speeds are sufficient to transmit the excess speed as fast as the wave pulses arrive. At extremely high frequencies, however, or at extremely low pressures (when the molecules must travel farther between collisions), only the faster molecules participate in the transmission process. As a result, the higher frequencies travel with greater speed.

PROPAGATION THEORIES

In the mathematical theories that have been developed for the transmission of sound in monatomic gases, the equations can be expressed in terms of two variables, r and K. The first of these, r, is a measure of the ratio of the mean frequency of molecular collisions to the sound frequency. In the Bureau's experiments, r was taken as the independent variable and was varied by changing the pressure while the frequency was held constant at 11 Mc/sec.

The other quantity, K, called the propagation constant, is a complex number; that is, $K = \alpha + i\beta$, where α is a measure of the attenuation, or rate at which the sound amplitude falls off with increasing distance, and β is proportional to the ratio of frequency to speed. Since the frequency was held constant in the present experiments, β could be considered inversely proportional to the speed.

Three theories of sound propagation in monatomic gases were compared with the Bureau's measurements. All of these are based on the assumption that the Boltzmann integro-differential equation for the distribution of molecular velocity is valid even for the enormous velocity and temperature gradients that occur in the experiments described. The first theory, which represents the classical approach, was originally developed by Kirchhoff, though it is better known as the Stokes-Navier theory in the form here used. The other two, known in the literature as Burnett theories, make use of modern developments in kinetic theory; they were developed by C. S. Wang Chang and G. Uhlenbeck. The first of these (referred to below as "the Burnett theory") is based on equations obtained from a velocity distribution correct in the second approximation. The other theory ("the super-Burnett theory") corresponds in the same way to the third approximation.

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[2] "Propagation of Sound in Rarefied Helium," by M. Greenspan, *J. Acoust. Soc. Am.*, Vol. 22, p. 568 (1950).

For small dispersions, the three theories are in agreement. When $r = 8$, the speed of sound should be about 1 per cent above its normal value. For helium at room temperature and standard pressure, this corresponds to about 100 Mc/sec and the mean collision frequency of the molecules is about 61 times the sound frequency. The increase in speed is about 10 per cent for $r = 3$, which corresponds to a mean collision frequency about 23 times the sound frequency, or to 260 Mc/sec at room temperature and standard pressure.

THE DOUBLE-CRYSTAL INTERFEROMETER

The Bureau's measurements were made with a double-crystal interferometer[3] of the type whose theory was described by W. S. Fry. The instrument was first built in 1950 and was used for studies of the propagation of 1 Mc/sec sound waves in helium. It was later modified to operate at 11 Mc/sec and several improvements in design were made. The changes led to better protection of the gases against contamination, elimination of mechanical cross talk, and improved control of electrical cross talk.

The essential parts of the device are two X-cut quartz crystal transducers for transmitting and receiving 11-Mc/sec sound waves. Electrical circuits are used for exciting the transmitter crystal and for amplifying the received signal and measuring its phase and amplitude. The attenuation and speed of the sound are determined from the changes in amplitude and phase of the received signal when the length of the sound path is altered by shifting the transmitter crystal.

Although called an interferometer, the instrument is not operated in the interference region, but under conditions that make the standing waves between the crystals practically negligible. Under such conditions the received phase varies linearly and the received amplitude exponentially with the length of the sound path. The amplitude is presented by a recorder having a logarithmic characteristic, and the phase by a conventional recorder. As a result, when the recorders are driven synchronously with the motion of the transmitter crystal, the slopes of the straight-line records give directly the attenuation and velocity of sound in the medium.

EXPERIMENTAL RESULTS

The measurements show that all five gases behave in substantially the same way at 11 Mc/sec. In the range for which dispersion is negligible (r greater than about 10) and for which the various theories are nearly indistinguishable, the measured values agree with the calculated ones very closely. For this range, then, the classical theory gives as good results as the modern theories.

[3] "An Eleven Megacycle Interferometer for Low Pressure Gases," by M. Greenspan and M. C. Thompson, Jr., *J. Acoust. Soc. Am.*, Vol. 25, p. 92 (1953).

For r less than 10, the Burnett theory showed the best agreement with experiment. This theory appears to give a satisfactory account of the velocity of sound for r greater than about 1, and of attenuation for r greater than about 0.5.

The fact that the Burnett theory agrees better with experiment than does the super-Burnett theory may at first seem surprising. However, even before the Bureau's measurements were made, it was suspected that the Burnett theories based on successively higher order approximations to the velocity distribution would alternately approach and recede from the correct result. The present measurements therefore raise a convergence question with respect to the method of expanding the distribution function.

A number of efforts have recently been made to improve the theory. A Burnett theory demands complex and troublesome manipulation of the non-linear Boltzmann equation; the final results are linearized by assuming the amplitude of the sound waves to be small. Being unsatisfied with this Wang Chang and Uhlenbeck and, independently, Mott-Smith have introduced the assumption of small amplitude and the consequent linearization directly in Boltzmann's equation. Unfortunately, this leads to a series development which is not useful for small values of r. Pekeris has attempted to correct this defect by resorting to a succession of approximations, each in turn the root of a polynomial of higher degree. The roots are obtained by numerical methods and the convergence is to be judged by the spread of successive results. Much interesting work on new methods of solving the Boltzmann equation has also been done at the N. Y. U. Institute for Applied Mathematics and Mechanics, though the application of this work to sound propagation is not yet available.

THERMAL RELAXATION

The Bureau's studies of monatomic gases are intended as part of a larger program to accumulate data on the dispersion of sound in various gases and mixtures of gases and to correlate these data with the available theories. Preliminary measurements have already been made in air, oxygen, nitrogen, and hydrogen. In such polyatomic gases and mixtures, another source of dispersion—thermal relaxation—is present in addition to that found in monatomic gases. This too is a consequence of non-uniformities in the gaseous medium.

In polyatomic molecules, energy is stored in the form of rotation or vibration (or both) as well as in the form of kinetic energy of translation. Under equilibrium conditions the energy is shared by these different modes in accordance with the law of equipartition of energy. During the passage of a sound wave, however, energy is transmitted first in the form of kinetic energy of translation; and a period of time must elapse before this is shared with the rotational and then with the

vibrational modes. In carbon dioxide, for example, it requires about 60,000 mean collision intervals for the energy to be transferred to the vibrational modes. Hence, as the ratio of sound frequency to collision frequency rises, the proportion of energy in the translational form increases and the sound wave is transmitted with correspondingly greater speed.

It was found in the Bureau's preliminary studies that rotational dispersion in oxygen and nitrogen occurs at frequency-to-pressure ratios so high that they overlap those associated with translational dispersion. A theory of the combined effect of the two sources of dispersion was therefore developed[4] so that the data could be properly interpreted.

Vibrational dispersion in carbon dioxide and other linear triatomic molecules has been thoroughly studied by a number of investigators. Rotational dispersion, however, such as occurs in oxygen and nitrogen, has only been touched upon because the relevant frequency-to-pressure ratios are beyond the range of hitherto existing equipment. The relaxation-time data obtainable from sound propagation studies of these components of air would have immediate application to practical shock-wave problems and high-speed upper-atmosphere flight.

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[4] "Combined Translational and Relaxational Dispersion of Sound in Gases," by M. Greenspan, *J. Acoust. Soc. Am.*, Vol. 26, p. 70 (1954).

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MEMBERSHIP

The following elections were approved at the Board of Managers' meeting on September 19, 1956:

June 1, 1956 to June 30, 1956

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Eliot B. Hanscom
Robert A. Shattuck
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W. C. Wingate

ACTIVE

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George W. Fewkes
Robert M. Grogan
Albert W. Holbrook
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A. C. P. Quimby
J. Ralph Satterthwaite
Erich Scheerer
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R. W. Struse

ACTIVE NON-RESIDENT

Barry J. Hildebrand

July 1, 1956 to July 31, 1956

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Frederick W. Johnson
Sheldon Kapustin
Abraham M. Kroll
Ross L. Leffler
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Frank W. A. Myers
Granger D. Schrader
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Robert P. Abrahams

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Samuel J. Herr
Frederic D. Justin
Ridgway Kennedy, Jr.
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Thomas J. Mancino
R. H. Peckham
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Robert B. Seidel
Marion F. Stone
Walter B. Thompson
Edwin A. Walter
Gregory S. West
Richard G. Wickes

ACTIVE NON-RESIDENT

Roy D. Holwager

Winfield W. Loose

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A. B. Davis '45
Horace F. DeAncona '53
Paul H. Dike '40
E. T. Downham '46
Arthur E. Espenshade '51
Jacob R. Fox, Jr. '42
Victor Russell Fritz '41
William A. Herr '35
Benjamin A. Lerten '46
Rudolph M. Lombardi '49
Edward W. Madeira '36
Ralph E. McConnell '42
R. M. Norstrom '53
Edward E. Reardon '49
Harry J. Schehm '36
Jack V. Scott '54
Frank M. Speakman '43
George Uetz '43
G. A. Warburton '47
David Weber '43
Paul B. Wendler '54
Herbert J. Wetherill '49

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

KAVANAGH, R. J.: The Application of Matrix Problems to Multi-Variable Control Systems

KRON, GABRIEL: Improved Procedure for Interconnecting Piece-Wise Solutions

GARDNER, GEORGE S., HARRY L. FAIGEN, GREGORY L. GIBSON AND WILBUR S. HALL: Researches on Corrosion and Inhibition. Reaction Velocity in the System Iron: Dilute Acetic Acid at 40° C.

CHANG, S. S. L.: Transient Effects of Supply and Connecting Conduits in Hydraulic Control Systems

STEINMAN, D. B.: The Design of the Mackinac Bridge for Aerodynamic Stability

PIPES, LOUIS A.: Analysis of Electric Circuits Containing Nonlinear Resistance

MARIN, JOSEPH A.: Mechanical Properties of Materials for Combined Stresses Based upon True Stresses and Strain

HODGE, P. G., JR.: Piecewise Linear Isotropic Plasticity Applied to a Circular Cylindrical Shell with Symmetrical Radial Loading

TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.

There is still a continuing and pressing need of contributions and bequests —both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.

Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

PETROLEUM

The newspaper reports upon recent happenings in the Near East have served to focus attention upon the oil situation in that part of the the world, and caused many people to realize the inadequacy of their knowledge of this important commodity, even in its domestic aspects. To most city dwellers, oil means very little more than liquid to be poured into an automobile.

An excellent opportunity to improve his or her knowledge of the location, extraction, and utilization of petroleum is offered to every Museum visitor. The presentation of this picture of an industry has been raised to a highly entertaining degree, even to that most repulsive (to the visitor) feature of all instruction, the statistics.

The miracle fluid which we call petroleum was formed in primordial slime millions of years ago, when shallow land-locked seas had swarms of marine life such as diatoms, algae, mollusks, and fishes now extinct. Some of these in fossilized form may be seen in the specimens of sand drawn from well drillings which are displayed at the Museum. Sediments gradually covered their dead bodies and chemical changes took place to form the oil which is now recovered from the pores of underground formations.

Petroleum began its modern history with Drake's success in sinking a well near Titusville in 1859, but remains of ancient civilizations show that man's use of oil antedates historical records. In the Euphrates Valley the Sumerians used asphalt as early as 3800 B. C. for embalming, calking boats, making roads and bricks. The floor of Nebuchadnezzar's temple made of stone set in asphalt is still intact. Petroleum was also used as a purge for man and beast.

In the United States, the first gusher was most unwelcome. While some unsuspecting individual was drilling for salt water in Kentucky about 1829, he developed the gusher, which promptly took fire, to the consternation of everybody concerned. Prior to Drake's well, petroleum production in this country did not exceed sixteen barrels annually. That well was sixty-nine feet deep, while recently an Oklahoma well reached a depth of almost four miles.

It is always difficult for non-technical people to visualize such a spectacular growth of industry without the aid of devices such as those used in the Museum display, and only by these means can the rich diversity of this particular industry be comprehended.

The principal use of petroleum in the early days of the industry was to provide kerosene for lamps. During this period gasoline was largely a waste product. With the advent of the automobile a demand for gasoline arose which challenged the productive capacity of the refining industry. The demand for gasoline expanded at such speed there seemed to be an impending shortage, and the industry was driven to increase the yield of gasoline from every barrel of crude. In the search for new sources of supply every modern science is brought into play.

Petroleum research only begins with the search for new sources. Intensive research is pursued every inch of the way. It has had a significant influence upon our food problem. The farmer is engaged in constant warfare with flying, crawling, biting, sucking, and boring forms of life which cause terrific crop losses and sickness among poultry, pigs, sheep, and cattle. Derivatives of petroleum have been made into deadly weapons to combat worms, insects, rats, and weeds. Other petroleum products stimulate life processes. Olefinic gases like ethylene induce growth for potatoes and citrus fruits.

The inventive genius of the petroleum chemist has developed processes for manufacturing glycerine and detergents or soapless soaps, from petroleum. He has also solved or alleviated other shortages by inventing plastics materials which are used in products ranging from paints to wallboards. The great significance of waterproofing material was unrecognized until the war came. But the Pacific tropics required these materials for shoes, clothing, tents, and packages. Before these were developed articles went to pieces in a matter of weeks or months. This problem was solved by copper naphthenates and microcrystalline wax derived from petroleum.

As a raw material, petroleum is a veritable treasure house in the lengthening of the span of man's life, and helps assure him of greater health and happiness than he has ever known. For this reason the Museum is proud to have the admirable Sun Oil Company's exhibit to offer to its visitors.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

LIQUID DENSITY METER DEVELOPMENT*

BY

C. W. HARGENS

It has been pointed out that hydrometers used in proofing whiskey have a number of disadvantages, and there has existed for some time the belief that some improvement might be realized through the use of more recent electronic methods.

An electronic liquid density instrument is being developed in the Electrical Engineering Division of the Institute Laboratories. Below is a description of the instrument in its present state of development. Typical accuracy data obtained through this initial experimental period are also indicated.

It is felt that this instrument may find application in various industries requiring the routine measurement of liquid densities. Such industries include: foods, oils, chemicals in general, as well as distillation.

The present instrument is about the size of a table model radio. It is powered from a 115-volt a-c. line. Samples to be measured are placed in a small glass jar, the vessel requiring approximately 100 cc. of liquid for proper functioning.

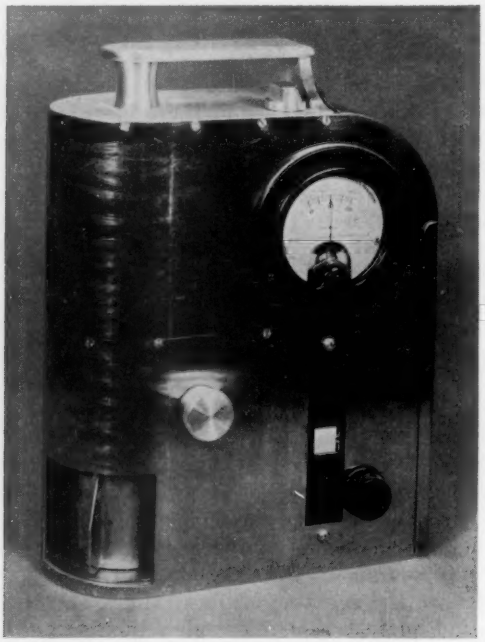
Once the container has been introduced into an opening in the side of the case, a knob is turned, actuating a miniature elevator which gently hoists the sample to a level at which the measuring float is properly immersed. When this has been done, the operator observes a zero-center, null-indicating meter while adjusting a second knob, the only other control save the on-off switch. The second knob simultaneously balances the instrument and drives a scale which is calibrated in density units. When balance is registered by the null indicator, the density may be read directly from the scale.

Since the sensitivity of the instrument is sufficient to indicate variations in density caused by changes in temperature, the readings would normally reflect the temperature of the sample. A thermostatically-controlled, immersion-type heater of large area has therefore been built into the apparatus to maintain a set temperature.

The density range for which this particular instrument was designed

* This work was sponsored by Louis Forman of Philadelphia. A detailed technical paper entitled "A Portable Liquid Density Instrument Employing Transistors" is being prepared by Mr. Hargens.

is from 0.8000 to 1.0000, with the desired reproducibility over this range being one part in 2000. The calibrated scale is sufficiently long to permit the resolution of these increments (0.0001 density). Likewise the null indicator, employing transistor electronics, is sufficiently sensitive to show a needle displacement of 10 divisions ($\frac{1}{2}$ in.) for a density difference of 0.0001. There should be no difficulty in increasing the range of the instrument, and, owing to its principle of operation, very little change in accuracy should result from such increases.



Transistorized instrument developed by the Laboratories for measuring the density of liquids.

Since transistors have been used throughout the electronics of the device, no warm-up time is required before a precise density reading can be obtained. Depending upon the initial temperature of the sample, a brief period (less than a minute) is required for the thermostat to stabilize.

The nature of the electrical output of this density measuring device is such that a continuously recording instrument may be readily attached. Automatic balance by means of an electronic servomechanism is now being worked out together with provision for the continuous record attachment.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, M.D., FOUNDER

WILLIAM G. BATT, D.Sc., DIRECTOR

THE LIPOPROTEINS OF STORED LIQUID PLASMA

BY

LAURA E. KREJCI, LUCILE SWEENY AND JOHN HAMBLETON

Room temperature storage of pooled liquid plasma for six months or longer has been found to remove almost completely the risk of transfusion reactions (1) and of transmission of the virus responsible for homologous serum jaundice (2). The clotting factors and complement disappear rapidly (3), but by electrophoresis the albumin and alpha, beta and gamma globulins show little detectable change in concentration short of two and one-half years (4); thereafter the gamma

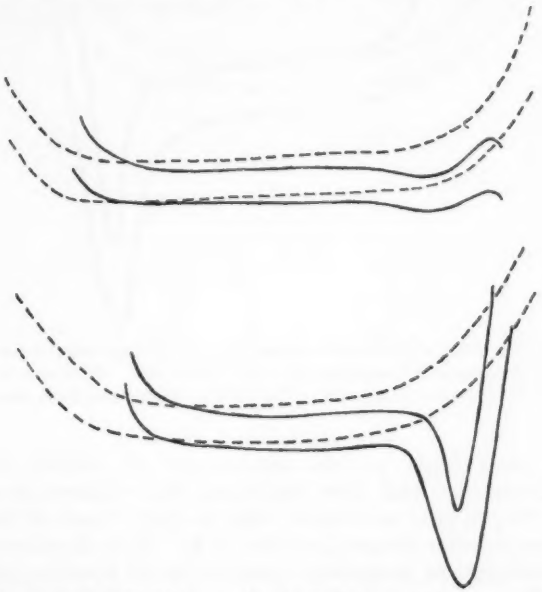


FIG. 1. Lipoproteins of liquid human blood plasma stored 19 months: ultracentrifugal flotation at density 1.063. (*a*) (*Upper diagram*) Component $S_f = 26$; 48,000 rpm. (*b*) (*Lower diagram*) Component $S_f = 6$; 10 minutes at 55,000 rpm. The plasma was prepared from blood collected in ACD, pooled into 2-liter bottles containing 200 ml. of 50% dextrose, stored at 37° C. for six months, and stored thereafter at 25–28° C.

and beta globulins drop in concentration (**4, 5, 6**). These losses appear to be complete at about five years (**4**), and there is little further change through ten to twelve years (**4, 6**). In the ultracentrifuge the plasma gives well-defined sedimentation boundaries after six years of storage (**6**). The available evidence seems to indicate that the proteins resisting hydrolysis or precipitation undergo relatively minor changes and display surprising stability.

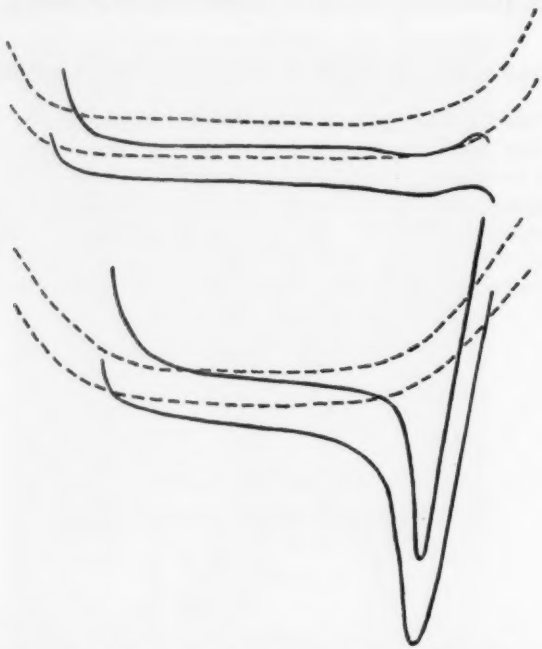


FIG. 2. Lipoproteins of fresh human blood plasma: ultracentrifugal flotation at density 1.063. (*a*) (*Upper diagram*) Component $S_f = 35$; 42,000 rpm. (*b*) (*Lower diagram*) Component $S_f = 7$; 9 minutes at 55,000 rpm. The plasma was prepared from blood collected in ACD.

Unlike most other protein components of plasma, lipoproteins cannot be frozen or dried from the frozen state without gross denaturation, and they appear to be susceptible to many types of denaturation not common to other plasma proteins (**7, 8**). It is therefore of interest that two well-defined lipoprotein components of flotation rates $S_f = 6$ and $S_f = 26$ were found in liquid plasma[1] stored nineteen months at room temperature when analyzed for lipoproteins by the technique of

[1] Generously supplied by Dr. Lloyd Newhouser of the John Elliott Blood Bank of Dade County, Miami, Florida.

Gofman (9). On the other hand, plasma stored six years at room temperature, then refrigerated six years longer, contained no detectable lipoproteins when examined by the same technique. It appears that lipoproteins, in spite of their susceptibility to denaturation, are stable to room temperature storage in liquid plasma for at least a year and one-half. Like the gamma and beta globulins, however, they may be lost when the plasma is stored for longer periods.

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PUBLICATIONS RECEIVED

The Editors wish to call attention to these new books which have been received in the JOURNAL office for review. Lack of space prevents publication of full reviews.

MUSICAL ACOUSTICS, by Charles A. Culver. Fourth edition, 305 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.00.

ABACS OR NOMOGRAMS, by A. Gift, translated by J. W. Head and H. D. Phippen. 225 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1956. Price, $12.00.

ORGANIC CHEMISTRY, by Louis F. Fieser and Mary Fieser. Third edition, 1112 pages, 6 × 9¼ in. Boston, D. C. Heath and Company, 1956. Price, $9.00.

ENGINEERING MECHANICS, by S. Timoshenko and D. H. Young. Fourth edition, 535 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.50.

BIOLOGY OF THE LABORATORY MOUSE, edited by George D. Snell. Unabridged and unaltered reprint of the first edition. 497 pages, illustrations, 6¼ × 9¼ in. New York, Dover Publications, 1956. Price, $6.00.

U. S. ARMY IN WORLD WAR II. THE TECHNICAL SERVICES. THE TRANSPORTATION CORPS: MOVEMENTS, TRAINING AND SUPPLY, by Chester Wardlow. 564 pages, illustrations, 7 × 9¾ in. Washington, Department of the Army, 1956. Price, $4.25.

GLASS, by G. O. Jones. Methuen's Monographs on Physical Subjects. 119 pages, illustrations, 4 × 6½ in. New York, John Wiley & Sons; London, Methuen & Co., Ltd.; 1956. Price, $2.00.

ELEMENTS OF PULSE CIRCUITS, by F. J. M. Farley. Methuen's Monographs on Physical Subjects. 143 pages, diagrams, 4 × 6½ in. New York, John Wiley & Sons, Inc.; London, Methuen & Co., Ltd.; 1956. Price, $2.00.

MINERALS & THE MICROSCOPE, by H. G. Smith. Fourth edition, completely revised by M. K. Wells. 148 pages, plates, 4½ × 7¼ in. New York, The Macmillan Company, 1956. Price, $3.00.

MEASUREMENTS OF MIND & MATTER, by F. W. Scott Blair. 115 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1956. Price, $4.50.

BOOK REVIEWS

CHROMIUM. VOLUME I. CHEMISTRY OF CHROMIUM AND ITS COMPOUNDS, edited by Marvin J. Udy. ACS Monograph No. 132. 433 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1956. Price, $11.00.

This is an ACS monograph covering the chemistry of chromium and its compounds. The first quarter of the book consists of a general section covering the history, mineralogy and geology, analytical chemistry, and health aspects of chromium. The remainder of the monograph is devoted to the physical and chemical properties of chromium compounds and their uses in industry.

The first section opens with a readable historical sketch of the type found in most books devoted to a single metal. However, in the following chapter the reader finds himself immediately involved in a mass of mineralogical detail concerning seventeen chromium minerals with such names as Halotrichite, Kaemmererite, Phoenicochroite, Stichtite and Fornacite. One does not relax even with the simplest named chromium mineral, chromite, when he reads the author's statement ". . . the mineralogical term chromite without adjectival modifier is used in the broad sense as a general name for all chromium spinellids except pure end-members." If the purpose of the monograph were to have a reference section for the mineralogist and geologist, another for the chemist, etc. then this chapter helps fulfill that purpose since the coverage is thorough and erudite. However, it is this reviewer's opinion that this chapter should have paralleled the others in Section I which were not directed to the historian or medical profession, but were of general interest.

One of the outstanding chapters in the book is that dealing with the relation of chromium to the health. There is a tendency for many of us to consider that such a common metal as chromium does not constitute a health hazard even when it is combined into some of its more hazardous hexavalent compounds. Many researchers or workers are not assessed of the health problems involved with the use of these metals until they have experienced some sort of injury. Anyone using this book as a reference is most likely to check to see if the problem in which he is interested or intends working on involves a health hazard.

Over one quarter of the book is devoted to the physical and chemical properties of compounds of chromium. This is an outstanding compilation and should serve as an invaluable reference to those interested in these compounds.

Separate chapters deal with the use of chromium chemicals in the various industries, namely: textiles, tanning, oil and gas, wood preservation, pigments and graphic arts. These chapters are mostly of general interest; the coverage is quite thorough and should be of interest to the manufacturer and uses of the compounds.

This volume will be of primary interest to the chemist and should serve as a valuable reference text for data on the physical and chemical properties of the chromium compounds. R. L. SMITH

RADIO ELECTRONICS, by Samuel Seely. 487 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.00.

Professor Seely, now head of Case Institute's Electrical Engineering Department, has prepared another general text on vacuum tube techniques as applied to communications. This is his third book dealing with about the same material, namely, amplifiers, oscillators, modulators, and detectors.

If one did not look at the publication date, it would be difficult to state the period in which this book had appeared. The only clue is perhaps in the problems and tube characteristics in the appendix. These refer to tube types popular fifteen or twenty years ago and now rather obsolete. Although still

practical, it might have been desirable to supply more contemporary information in the interest of timeliness.

Although most of the book duplicates the earlier works of Terman, Everitt, and Reich which were published several decades ago, a few wartime circuit developments have been added. Also several circuit outgrowths of the period in which commercial television was being developed are presented. For example, Wallman's analysis of stagger-tuned, multistage amplifiers has been included under "Tuned Potential Amplifiers." Also the "Twin-T" circuits for obtaining narrow band-pass characteristics at low frequencies are covered in the same chapter.

It is desirable for the professional engineer to have sufficiently broad training so that when various principles are discussed in terms of their discoverers he will recognize them without having to ask his associates to stop and explain. For instance, when Richardson's and Dushman's equations are mentioned, it should not be necessary to explain to the electronics engineer that we are referring to the mathematical relations describing electron emission from hot metals. Professor Seely tends to omit such information, giving only the bare facts. It is hoped that this is not a trend which others will follow because, besides giving credit to authorities in the field, a valuable saving is achieved by identifying a principle by a name.

Aside from the several additions mentioned, this book is conventional in all respects, following the usual tube and circuit analysis via the algebraic process. No mention is made of transistors, and perhaps this is wise, for the book does give the impression of sticking closely to a well-defined subject and treating it with thoroughness. There is no doubt that a good basic course on vacuum tube amplifiers and their common communications applications could be successfully taught with this volume serving as the text.

C. W. HARGENS

THERMAL POWER FROM NUCLEAR REACTORS, by A. Stanley Thompson and Oliver E. Rodgers. 229 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $7.25.

In the fissioning of the uranium or thorium atom energy is released. A single pound of these elements fissioned in a reactor will produce the heat equivalent of 1400 tons of coal. This makes the production of energy by the fission method a highly attractive enterprise. Add to this the fact that the energy reserves in fissionable material are approximately 25 times that of all the other fossil fuels combined and we see that the energy production by fission is not only an attractive method but a necessity if our energy conscious civilization is to continue at its present rate.

If this is the case then why don't we go to production of energy from nuclear reactors? Indeed why don't we? The answer to this simple question is contained in a book of over 200 pages written by two eminent engineers: A. Stanley Thompson and Oliver E. Rodgers.

In *Thermal Power from Nuclear Reactors* the authors have shown that the answer is simple but the details of putting this device into operation is of inordinate complexity. They begin by setting down the basic reactions on which reactors operate. Explanatory material, in some cases much too elementary for a work of this type, is included. By the end of chapter one the stage is set for the design of a reactor.

Because neutrons behave in curious ways in identical situations the authors show that it is almost impossible to design "criticality" into the reactor. Engineers must arrive at this critical mass of fissionable material by making assumptions on the source distribution of fission neutrons and their energy spectrum. The numerical solution is made and compared with the assumption. The two will not coincide so that an iterative process is employed to arrive at the critical fissionable mass to be used in the reactor.

Control of a reactor is of paramount importance. A runaway reactor is in essence a slow atomic bomb which can scatter viciously radioactive materials over wide areas and into water supplies. How is the control designed? Of this the authors write: "Some difficulties exist in the design of the control system for a nuclear reactor which stem primarily from the fact that reactors are so expensive that they are built in limited quantities and in general cannot be built for such purposes as testing control systems. Most of the estimates of the operating characteristics of a

reactor which are pertinent to the design of a control system are based on mathematical analyses. These operating characteristics are so complicated that it is not practical to attempt to write mathematical equations to describe their behavior completely and even if they were written they would be too complicated to be feasible for solution." Actually the equations to be solved are simplified approximations justified only by empirical methods. Control systems can be and are built but this is not a simple task and their successful operation is a tribute to the technical skills of the designers rather than of their analytical abilities.

Reactors operating at high power levels emit intense radiation. This radiation is deleterious to mechanical and electronic materials as well as the human body. How is this radiation prisoned to prevent the destructive and harmful effects? The answer we find in the shielding of reactors which comprises Chapter IV.

Reactor materials such as moderators and reflectors for the interior of the reactor are covered in Chapter V. Radiation can and does damage reactor materials and the authors indicate that significant advances must be made in the near future in this field to permit the engineer to predict performance of metals under intense radiation.

The last three chapters deal with the power generated in the reactor. The engineer encounters a critical problem in this field for the thermal stress in a reactor is high for we are dealing with a large temperature gradient and not all metals are suitable for use in a reactor. The problem of extracting heat from reactors is still but little understood and again the authors have performed magnificently in presenting the various concepts. The last chapter deals with conversion of this heat to useable power.

The authors should be justly proud of this book for it is a pioneering effort. In the immediate future there will be many reactors built both here and abroad and with these installations will come the definitive knowledge which will make reactor design more of an exact science. So thorough is this work that the men who build these new reactors will do well to use *Thermal Power for Nuclear Reactors* as their bible. Today it is quite definitely a bible though its life is destined to be short. I. M. LEVITT

PHYSICS AND MATHEMATICS. VOLUME ONE, PROGRESS IN NUCLEAR ENERGY SERIES ONE, edited by R. A. Charpie, J. Horowitz, D. J. Hughes and D. J. Littler. 398 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc.; London, Pergamon Press Ltd.; 1956. Price, $12.00.

This volume is the first of a series concerned with progress in the various fields of nuclear energy. It is to be followed by volumes in seven other fields, which include process chemistry, technology and engineering, biological and medical sciences. According to the plan for this series, the first volume in each field will consist of review articles on topics discussed at the Geneva Conference on the Peaceful Uses of Atomic Energy, plus a few of the papers presented at the conference. Subsequent volumes, not necessarily issued at regular intervals, will cover more recent progress.

This series is supported by an editional advisory board consisting of experts in nuclear energy from eight different countries, and is intended to summarize the work of scientists in all countries. The concept of international scientific cooperation in this area of nuclear energy, initiated at the Geneva Conference, is evidently the motivation for this series. For this reason, this endeavor should be commended wholeheartedly.

Despite its title, "Physics and Mathematics," this first volume is primarily a collection of review articles on experimental techniques and measurements in neutron and reactor physics, with two theoretical papers included. The level of difficulty and general structure of these articles is quite similar to those contained in the Annual Review of Nuclear Sciences. The eleven articles are written by experts in each particular topic, and are therefore of considerable value to those interested in neutron and reactor physics. According to the foreword by Prof. V. F. Weisskopf, these articles were intended to evaluate and critically compare the results presented at the Geneva Conference concerning the neutron physics required for reactor design, and the application of this knowledge to the practical arts of critical assemblies and reactor calculations. This goal was achieved admirably in the first two articles, which collect and carefully compare the results of all laboratories in

various countries on the cross-sections and excitation functions of U^{233}, U^{235} and Pu^{239} under neutron bombardment. These articles are particularly valuable, since they present the data from the Geneva Conference which may not be readily accessible to all those interested, and in addition, they include some data made available since the conference. These two articles are followed by a shorter article by Prof. H. A. Bethe on the theoretical analysis of neutron resonances in fissile materials. Another article of similar interest is that by G. R. Keepin on delayed neutrons, which provides a comprehensive survey of existing measurements.

There is a considerable variation in the presentation of the remaining articles, some of which are almost completely devoted to work done in the United States. A review article on the techniques for measuring elastic and non-elastic neutron cross-sections by five Los Alamos physicists seems to be too brief for the number of topics discussed, with very little attention paid to the work done in other countries. The last four articles, which comprise nearly half of the volume, deal with the properties and the theoretical study of critical assemblies of different types. These include a Russian theoretical paper on heterogeneous methods for calculating reactors, by S. M. Feinberg, and a review article on the physics of fast reactors by three scientists at Harwell.

This volume represents an excellent beginning for the series. The value of the critical comparisons of the data from the Geneva Conference is enhanced by the surprising promptness in publishing this volume. This volume will be a valuable reference work for all laboratories and industries interested in reactor physics; however, its price is somewhat prohibitive for the individual scientist.

D. M. VAN PATTER

THEORY AND PRACTICE OF LUBRICATION FOR ENGINEERS, by Dudley D. Fuller. 432 pages, diagrams, $5\frac{1}{4} \times 9$ in. New York, John Wiley & Sons, Inc., 1956. Price, $10.50.

For the first time in the ancient art of lubrication, a book has appeared which is designed specifically for the student. It is organized on the basis of a series of lectures given in Professor Fuller's graduate course in lubrication at Columbia University. Professor Fuller is recognized in the academic world as being one possessed of an inordinate ability to organize his lecture material and to present it to his students in an easily understandable manner. In transferring his words to paper, Professor Fuller has lost none of the above-mentioned ability. The book is one of the most easily understood texts in any field; nevertheless, it is comprehensive in scope and presents the basic theories in their entirety.

The first nine chapters are devoted to thick film bearings. These are bearings in which the bearing materials are separated by a film of lubricant thick enough for the bulk viscosity of the lubricant to be an important variable in design. Included in these chapters, therefore, are comprehensive sections dealing with the fundamentals of fluid flow and viscosity. Hydrodynamic and hydrostatic bearings using either compressible or incompressible fluids are discussed quite fully.

Chapters 10 and 11 present in a generalized and systematized manner, the accepted theories of dry and boundary friction. The effects of surface cleanliness, pressure, velocity, temperature, moisture and roughness are discussed. Solid and liquid lubricants are major topics for discussion in these chapters.

The last chapter is devoted to a discussion of bearing materials. Conventional materials as well as some of the newer ones (such as ceramics) are included.

Two of the best features of this book are the numerous examples presented to illustrate each theory, and the long list of references at the end of each chapter. No less than 296 references are included to aid the student in delving further into any aspect of this field.

In summary, it should be said that this book fills the need for a text which bridges the gap between theoretical and applied lubrication and is ideal for classroom or individual study. Professor Fuller is to be congratulated. GEORGE ROBINSON

A TEXT-BOOK OF PRACTICAL ORGANIC CHEMISTRY INCLUDING QUALITATIVE ORGANIC ANALYSIS, by Arthur I. Vogel. Third edition, 1188 pages, diagrams, $6 \times 9\frac{3}{4}$ in. London, Longmans, Green and Co. Ltd., 1956. Price, 60/ ($12.00).

Here, in one textbook, is probably the most complete and up to date presentation of practical organic chemistry that has been written to date. Dr. Vogel presents, in this massive volume, not only an excellent series of organic syntheses ranging from those suitable for the beginner to ones complex enough for the advanced or graduate student, but also a well organized discussion of the theory and use of the various experimental techniques, and a better than average procedure for the qualitative analysis of organic compounds.

Despite the length of the book (over 1100 pages) it maintains, throughout, an easily read and followed style without becoming merely a tabulation of recipes in cook book fashion. Further, the book teaches the student not only to perform syntheses, but also to know what is occurring in specific reactions, and, equally as important in the long run, why the recommended equipment and procedures are best suited for any given synthesis. In addition, since not only are classical syntheses and procedures included but also the most current work and equipment, the value of the volume is doubled.

The book itself is divided into twelve chapters and a ten part appendix. The first two chapters deal with theory and practice of general organic chemical techniques such as distillation, crystallisation, and fractionation. The next three chapters present syntheses involving aliphatic, aromatic, heterocyclic, and acyclic compounds using mostly traditional reactions, but with somewhat out of the ordinary compounds used in the syntheses. An example of this is in the oxidation of a methyl ketone to an acid where mesityl oxide is the ketone and is converted to beta-beta dimethylacrylic acid. Chapter 6 is titled "Miscellaneous Reactions" and contains such things as the Wohl-Ziegler reaction (allyl position halogenation), epoxidation of ethylenes, ring enlargement by diazomethane, and ketone and keto-ester syntheses using organo-cadmium compounds. The next two chapters give preparations of organic reagents, dyestuffs, and indicators, and chapter 9 presents syntheses of physiologically active compounds such as the barbiturates and sulfa drugs. The preparation of thermosetting and thermoplastic resins and synthetic rubber is treated briefly in the next chapter and the final two chapters cover qualitative organic analysis and techniques for semimicro scale preparations in that order. In the appendix the use of Beilstein and other organic chemistry references is discussed in detail as well as subjects such as laboratory first aid, and applications of infra red and ultraviolet spectra to organic chemistry. The foregoing description of the contents and organization of this book, while somewhat lengthy, is included to strongly point up the completeness of this work.

In the opinion of the reviewer this book will very quickly become one of the classics among preparatory organic chemistry texts, and on this basis it must be highly recommended to students, teachers, and even research workers in the field.

DONALD H. RUSSELL

BOOK NOTES

COMBUSTION PROCESSES. VOLUME II OF HIGH SPEED AERODYNAMICS AND JET PROPULSION, edited by B. Lewis, R. N. Pease and H. S. Taylor. 662 pages, diagrams, 6¼ × 9¼ in. Princeton, Princeton University Press, 1956. Price, $12.50.

This second volume in the series on High Speed Aerodynamics and Jet Propulsion deals with the complex problems of combustion in jet engines. Five sections are devoted to these aspects of combustion: thermodynamics, chemical kinetics, flame propagation in gases, combustion of liquids and solids, and detonation processes. The final section is on the production of energy by nuclear reactions. Each section is composed of one or more papers on a particular phase of the subject, written by qualified experts.

The increasing importance of jet propulsion engines makes this work particularly valuable to combustion engineers. The delay of nearly five years between receipt of the manuscripts and final publication is to be regretted, for had the volume appeared in its proper sequence in the series, it would have been more useful and more up-to-date.

HISTORY OF ANALYTIC GEOMETRY, by Carl B. Boyer. 291 pages, diagrams, 6½ × 9¼ in. New York, Scripta Mathematica, 1956. Price, $6.00.

The author has gathered together in scholarly fashion the history of analytic geometry and has integrated the hitherto rather scattered earlier histories. Parts of the book have been previously published in *Scripta Mathematica*. Four chapters deal with the ancient Egyptians and Babylonians, through the early Greek contributions, the Middle Ages and the Renaissance, giving a good background for the "modern" period beginning with Descartes and Fermat. Criticisms of the "new geometry" are commented upon in a chapter entitled "The Age of Commentaries." The remainder of this fine history is devoted to the development of the calculus and the contributions of Newton, Euler, Lagrange, Ampere, Cauchy and lesser figures. A fifteen-page annotated bibliography and an index complete the work.

THE PETROLEUM REFINERY ENGINEER'S HANDBOOK, by J. F. Strachan. 168 pages, illustrations, 7⅓ × 9¾ in. New York, Philosophical Library, 1956. Price, $15.00.

This practical book is a revised version of a 1947 handbook entitled "Practical Inspection of Oil Refinery Equipment." The many new processes which have been developed in the last decade made it advisable to bring the previous book up-to-date. Since the first edition was privately published, perhaps not too many refinery engineers and inspectors are aware of its usefulness.

The material covers plant inspection (organization, methods and equipment), inspection procedure (cracking, distillation, refining, etc.), corrosion and protection of oil refinery equipment, calculations and general safety measures.

NUCLEAR METALLURGY, IMD Special Report Series No. 2. 94 pages, illustrations, 8½ × 11 in. New York, Am. Inst. of Mining & Metallurgical Engineers, 1956. Price, $3.75 (paper).

Under the above title, the AIME has published the proceedings of a Symposium on Behavior of Materials in Reactor Environment, held in February, 1956. Six papers by eight authors comprise this useful and practical contribution to the new field of reactor metallurgy. Interested readers will find the lists of references helpful, since they gather under one cover most of the scattered literature references in the field.

RESISTANCE WELDING, prepared by American Welding Society. 163 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1956. Price, $4.50.

In this handbook for resistance welding are covered all six resistance welding processes. New techniques such as slope control and welding of aluminum are described, along with processes, equipment, precautions, quality control, inspection and testing. Prepared by the Resistance Welding Committee of the American Welding Society, the handbook is concise, clear and complete.

"Honeycomb" Storage Unit.—An inch-square "honeycomb" developed by a General Electric scientist will store up nearly a million bits of information. Smaller electronic computers with bigger memories are among the anticipated applications.

Dr. Harold R. Day of the General Electric Research Laboratory recently described his new information storage tube at an electronics conference. At the heart of the tube is a thin sheet of glass in which small holes have been etched and then filled with metal. In practice, information is written onto one side of the honeycomb by an electron-beam scanning method similar to that used in television. A "reading gun" picks up the information from the opposite side. Early laboratory models have permitted storage for several minutes and Day is confident the time can be extended.

Since the holes in the honeycomb are spaced 500 to the inch, each square inch has 250,000 individual storage cells—and each cell will recognize at least 10 different levels of intensity from the writing gun. Logarithmic calculations increase to more than 800,000 the number of bits of information that can be stored at one time (250,000 times log to the base 2 of 10).

Possible applications for the new storage tube in addition to computers include television cameras and "scan converters" in which radar information is collected and then displayed on an ordinary television screen.

The "mosaic" principle has long been recognized as ideal for storage tubes, but Day's device is believed to be the first developed with sufficient resolution and ruggedness to be suitable for most applications.

Day also has solved the problems of "leaking" and "smearing" frequently encountered when individual storage cells are placed so close together. Each storage cell "remembers" by building up charge in the capacitance formed between the recessed metals plugs and the conducting layers deposited on both surfaces of the glass. The glass honeycomb itself acts as an insulator between the plugs and the surfaces layers—and between the plugs themselves.

Another advantage of Day's system is that the problem of registration is eliminated because the surface layers and the recessed metals plugs are inherently "lined up." Because the spacing between elements is rigidly maintained, the entire structure is very rugged and the usual problem of microphonics—electronic noise caused by mechanical vibration—does not arise.

Two methods of reading the information are available. In one method the electron beam erases the screen as it picks off the stored information; in the other the information can be left for re-reading or to be altered or added to by the writing beam.

Underwater TV Camera.—A new development in underwater television which is expected to have wide application in salvage operations, submarine engineering, marine biology, oceanography and, probably, in underwater intelligence in the event of another war, has been announced by Pye Ltd. of England.

This is a small hand-held underwater television camera—the smallest and least expensive yet produced, according to the makers. It enables diving operations to be effectively

supervised from above the water by expert observers grouped around a large screen.

With accurate visual information displayed before them, and by means of underwater loudspeaker equipment (available as an accessory to the camera), intelligent direction can be given the diver. With a pair of divers operating on a complex project even better results can be obtained, since one can hold the camera while the other follows instructions from above.

Provision is made for pictures reproduced on the screen to be easily photographed.

The small version of the camera—a 12-in. sphere weighing 38 lb. on land and buoyant in the water—is intended for operation down to a depth of 250 ft. However, to provide an adequate safety margin, the container has been designed to withstand a water pressure of 220 psi., corresponding to a depth of 500 ft.

Two handles fitted to the spherical camera can be removed and replaced by weights or lamps so that the unit can, if required, be used without a diver.

All camera adjustments are carried out from the control unit above water; the diver's only concern is to position the camera correctly. Electronically, the machine is similar to Pye's industrial TV camera.

A larger version has also been designed to operate at depths down to 3000 ft. This one, encased in an aluminum sphere 19 in. in diameter, is also weightless in water and can either be held in a diver's hand, suspended by a cable from a moving ship, or propelled by an electrically-operated cradle.

There is therefore nothing, according to the makers, to prevent this camera being launched from a submerged submarine and operated by remote control. Presumably it could be of considerable service in surveying enemy underwater installations in wartime, as long as it could operate undetected. At the finish of an operation it can be withdrawn to a housing on the hull.

"Mechanical Ostrich."—Details of tests on a new "mechanical ostrich" called the Lift-A-Loft, designed to ease building maintenance and "overhead" repair problems, recently were released by the Naval Supply Center in Oakland. The Oakland depot conducted the tests in a pilot study of "Lift-A-Loft" for the Bureau of Supplies and Accounts.

Tests show the machine, an all-steel unit with a triple-hydraulic extension capable of boosting its worker's platform to a maximum height of 27 ft., allows one man to perform overhead maintenance and repair jobs in one-fourth to one-fifth the time formerly required by at least two men.

Powered optionally by a 24-volt battery on a four-cylinder gasoline driven 24-volt motor generator, both the lower and upper units of the 9000-lb. "Lift-A-Loft" can be entirely controlled by one workman stationed on the machine's platform.

Machine safety factors include deadman controls operated by the foot, plus the fact that the lift will automatically halt when being raised if any obstacle is encountered.

The machine's platform can slide backward or forward a maximum of eight feet, allowing its worker to reach third-floor levels even when they are directly over unmoveable floor machinery.

Among the many uses contemplated for the device are: washing and relamping the lights and reflectors of buildings; painting of ceilings, in-

teriors and exteriors and trim; and repair and inspection of water sprinkler pipes. In a recent test at NSC Oakland, warehouse illumination was increased by 50 percent (according to "before" and "after" light intensity readings) when the machine was used in washing lofty lights and reflectors.

An 80-gallon "balloon" in the lower unit of the Lift-A-Loft serves as ballast and assists in the washing operations. Water is pumped from the balloon into three adjoining two-gallon bins on the platform at the worker's finger tips. Electrical outlets on the platform, which will easily support 500-pounds, are provided for a 24-volt vacuum cleaner, drills, motors, or extension lights.

Certain modifications will be made on the machine prior to Navy acceptance. Its seven wheels will be increased to 13 and hard rubber will be used in place of the formica finish on four of them. Their diameter also will be increased to assist in outdoor use of the machine.

Navy to Photograph Hurricanes with Rockets.—Tests to determine the feasability of rocket nose cones containing cameras have been completed in the vicinity of Wallops Island, Virginia, by destroyers and aircraft from the Atlantic Fleet, the Navy announced recently.

This dress rehearsal of recovery techniques is the first phase of a project to photograph hurricanes with rockets, under sponsorship of the Office of Naval Research and the Naval Research Laboratory. This project, part of the National Hurricane Research Project established under sponsorship of the U. S. Weather Bureau, will photograph the entire hurricane cloud system from an altitude of approximately 100 miles.

During the East Coast hurricane season this fall, four two-stage rockets will be launched under the direction of the pilotless aircraft research division of NACA at its facilities at Wallops Island. The rockets will be equipped with cameras and air and seaborne location and recovery devices. Floating rocket nose cones containing the cameras will be disengaged from the rockets and parachuted to the sea surface. Ships from Destroyer Flotilla FOUR will recover the cameras.

A two stage rocket, the NIKE-CAJUN, will carry the cameras, parachutes, and locating and recovery instruments to approximately 100 miles altitude. Pictures taken from this altitude will have a radius of about 800 miles.

After the films are recovered and developed, broad aspects of the storm will be studied. Analysis of the photographs will enable scientists to add to their knowledge of hurricanes.

Monel Lightning Rods.—It has been found that lightning rods may become useless in as little as eight years when used to protect chemical and power plant stacks. It is not the weather but the stack gases that do the damage. Rods with exposed sections made of Monel nickel-copper alloy are often used for this reason as this alloy has the necessary corrosion-resistance, strength and electrical properties to equal the life of the stack.

Work on "Re-Entry" Problem.—Scientists of Avco Manufacturing Corporation are solving the thorny problem of atmospheric re-entry, a major obstacle that has stood thus far in the way of developing an intercontinental ballistic missile, by means of a "scientific short-cut," it has been

announced by Dr. Lloyd P. Smith, Avco vice president and president of the company's Research and Advanced Development Division.

Avco scientists and technicians at Everett, Mass., and Stratford, Conn., have been actively engaged in the intercontinental ballistic missile (ICBM) program for some time. The announcement of Avco's participation was made at the Air Force Association convention in New Orleans by Brigadier General Don R. Ostrander, Assistant for Guided Missiles, Headquarters, Air Research and Development Command.

The company's job has been to develop the vital nose cone for king-size missiles and at the same time to solve the problem of the missiles' re-entry, a problem that has plagued scientists since the intercontinental ballistic missile program first got underway.

Re-entry is the point at which the missile would re-enter the atmosphere and plunge to its target after traveling thousands of miles at supersonic speed through space. The scientists' problem is to overcome the enormous air friction which would burn up the missile in the same way that most meteors burn up and vaporize as they fall toward the earth.

The problem is being solved at Avco's Research and Advanced Development Division, said Dr. Smith, through the use of new scientific methods in the laboratory, which are expected to save the huge amounts of time and money that would be required for actual missile flight tests.

The work on Avco's nose cone research and development has involved almost every field of science, ranging from high temperature gas dynamics to aerothermodynamics and metallurgy.

Over-the-Floor Extension Cord.—Called Electriduct, this new over-the-floor electrical extension duct for offices, homes and places where an electrical outlet is needed in the middle of the room is made of rubber. It is stumble-proof and unobstructive; office equipment on casters rolls over it easily. One end is plugged into a wall outlet and the other end has a two-way receptacle. The duct is $2\frac{3}{4}$ in. wide at the base, which rises from a feather edge to an apex of $\frac{7}{16}$ in. Ribs on the underneath side prevent slipping on the floor and the enclosed wiring is safe from moisture. The complete duct system ready to plug into a wall outlet is available in standard 4, 5, 6 and 10 ft. lengths. Special lengths are also available for custom installations. Electriduct is a new product of Ideas, Inc., of Laramie, Wyoming.

"Red Hot" Electronic Devices.—The penetration of a critical temperature barrier in the development of guided missiles and supersonic aircraft was dramatically demonstrated recently when General Electric scientists exhibited laboratory models of revolutionary electronic devices and circuits operating literally "red hot." The achievement was described as a major step toward overcoming the inability of electronic controls to withstand the skin-sizzling heat generated by air friction at extreme speeds.

An equally important "plus" feature of the new electronic circuits is their ability to operate for long periods while exposed to intense nuclear radiation in an atomic reactor.

Defense Importance

"Tremendous defense implications" were attributed to the heat-resistant, radiation-tolerant electronic devices

by Dr. C. G. Suits, GE vice president and director of research. He said the rapid progress reflected is the result of "extraordinary teamwork" among the General Electric Research Laboratory at Schenectady, N. Y., nearly a dozen other GE departments and laboratories, and various U. S. defense agencies.

"Radically new electronic concepts incorporated in these components and circuits represent a crucial breakthrough of the 'thermal barrier' and give new impetus to the nation's guided missile, satellite, and supersonic plane programs," Dr. Suits said. Among the advantages predicted are extensive weight reductions made possible by eliminating bulky, power-consuming cooling equipment from missiles and aircraft.

Variety of Components Used

Inside glowing electric furnaces and surrounded by blazing blow-torches, General Electric's new vacuum tubes, capacitors, resistors, transformers, inductors, wires, printed circuit boards —and even an electric motor—were operated at temperatures ranging from 900 to 1500° F. Metals such as titanium and special laboratory-designed ceramics play important roles in the design and construction of high-temperature components.

Dr. Suits emphasized that many electronic components other than those demonstrated are being and will be developed by General Electric. "These new basic concepts can be widely applied in almost every facet of the electronics industry," he said.

Nuclear Radiation Tests Described

In successful tests also announced by GE, electronic assemblies have been operated at high temperature for more than 1000 hr. in the neutron flux of the graphite reactor at the Oak Ridge National Laboratory.

"Due to the exceptional military demands of World War II, several components were developed up to 1945 that could operate at 300° F.," Dr. Suits said, "but the over-all reliability limit was, and in much of present-day equipment still is, about 200° F. The recent emphasis on guided missiles and supersonic aircraft has greatly increased the requirements, but until now extensive research and expenditure has produced only a few components capable of operation at 500 to 600° F."

"Now," he added, "in one dramatic step the combined efforts of General Electric scientists and engineers have produced not just individual components but even some basic circuits capable of operating at several hundred degrees higher than any individual component previously available."

"Hi-fire" Phonograph

To help show the versatility of the high-temperature electronic components, GE Research Laboratory scientists demonstrated a "hi-fire" phonograph with the amplifier glowing brightly in a 1500°F. oven. There was sufficient volume to fill the large demonstration room and surprising fidelity in reproducing the "hot jazz" record used in the demonstration. Also shown was another circuit that picked up and amplified a radio signal even while the equipment was in the direct flame of three blowtorches.

Other less dramatic, but scientifically significant, demonstrations included two multivibrator (electronic pulsing) circuits operating in electric ovens at temperatures ranging from 1000 to 1500° F. Both of these demonstrations involved circuits using

vacuum tubes, resistors, capacitors, and print-boards, and in one of the demonstrations "heaterless" tubes were used to show that at high temperatures there is no need for the power supply normally required to heat the filaments in the vacuum tubes.

Haller Discusses Missile Requirements

In his discussion of the defense importance of high-temperature electronic components, Dr. George L. Haller, General Manager of GE's Defense Electronics Division, described the problems surrounding the development of guided missiles including the so-called "ICBM" (intercontinental ballistic missile).

"Within the broad field of defense products," Dr. Haller said, "guided missiles provide one of the most challenging applications for high-temperature components. The electronic circuits, resistors, amplifiers, transformers, and tubes in a guided missile must be designed to operate reliably in two sources of heat: internal and external. The multitude of tasks required of a guided missile, combined with the all-important necessity of keeping weight and unused space at an absolute minimum, results in a severe concentration of many electronic components in a small volume."

"The ICBM is a classic example of an application of high-temperature components to the problem of external heating," he said. "The ICBM must descend from extreme altitudes at velocities of about 10,000 miles per hour and re-enter the dense layer of air surrounding the earth. Within this envelope of air, friction between the ICBM and the air around it generates heat equal to that heat generated by a half dozen or more of the world's largest steam turbines. If not dissipated, this heat rise can make ICBM electronic components either malfunction or fail."

Importance to Aircraft Nuclear Propulsion Stressed

The new components' tolerance to nuclear radiation was described as an important "extra feature" by G. W. Newton, manager of application engineering at the Aircraft Nuclear Propulsion Department.

"The Aircraft Nuclear Propulsion Department for the past two years has made an extensive effort to develop power plant components tolerant to nuclear radiation," he said. "During the course of the development, we found that many materials that are susceptible to damage by heat at temperatures less than 500° C. are also susceptible to damage from nuclear radiation. Now, with this demonstration of nuclear radiation tolerance, one of the technological advances needed to better meet the objectives of the Aircraft Nuclear Propulsion Department was at hand."

Newton described in some detail the tests in which electronic circuits were operated for over 1000 hours in the graphite reactor at the Oak Ridge National Laboratory.

"These tests constitute a feasibility demonstration of significant progress," he said. "We are well on our way, but there are many other technological advances which must be made to fulfill our programs for the U. S. Atomic Energy Commission and the U. S. Air Force. These advances will also require coordinated effort in the varied technological areas encompassed by the diversified operations of the General Electric Company."

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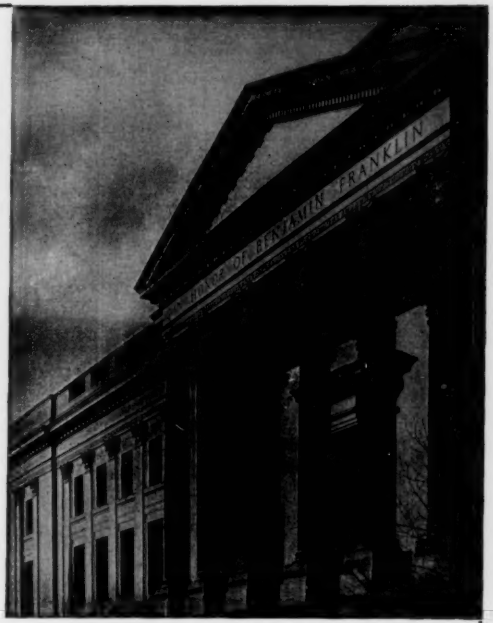

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